Image credit: LIGO/T. Pyle

Gravitational Wave Astrophysics

Pablo Marchant

Part 2: Ground based interferometers

Outline

Today

- History of the field
- Types of detectors
- Types of sources
- Current state of the field
- Future advancements

12/5

- Ground based interferometers
- Production of GWs from compact object binaries

19/5

- Parameter estimation from observed compact object coalescences
- Astrophysics of observed GW sources

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The technical challenge

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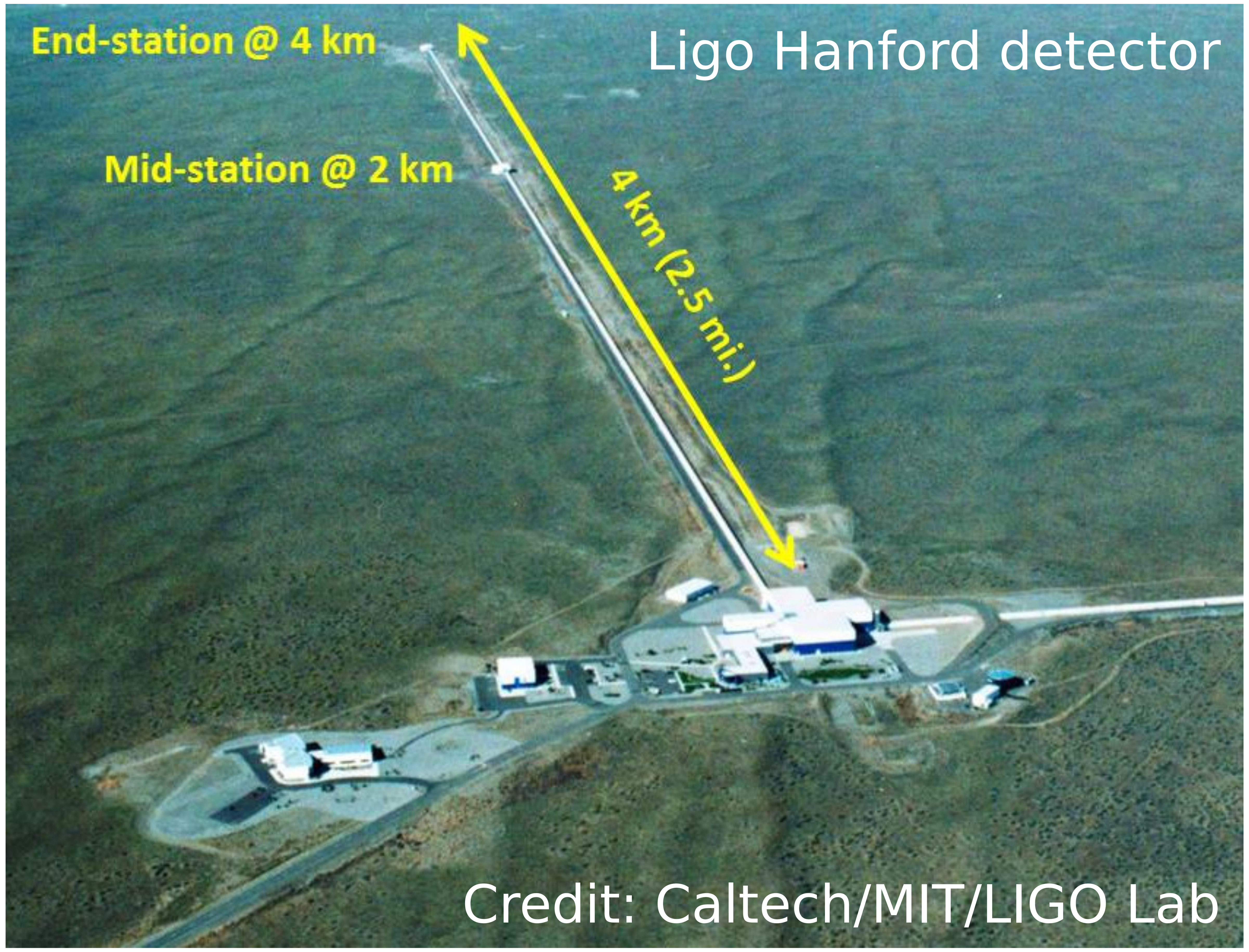

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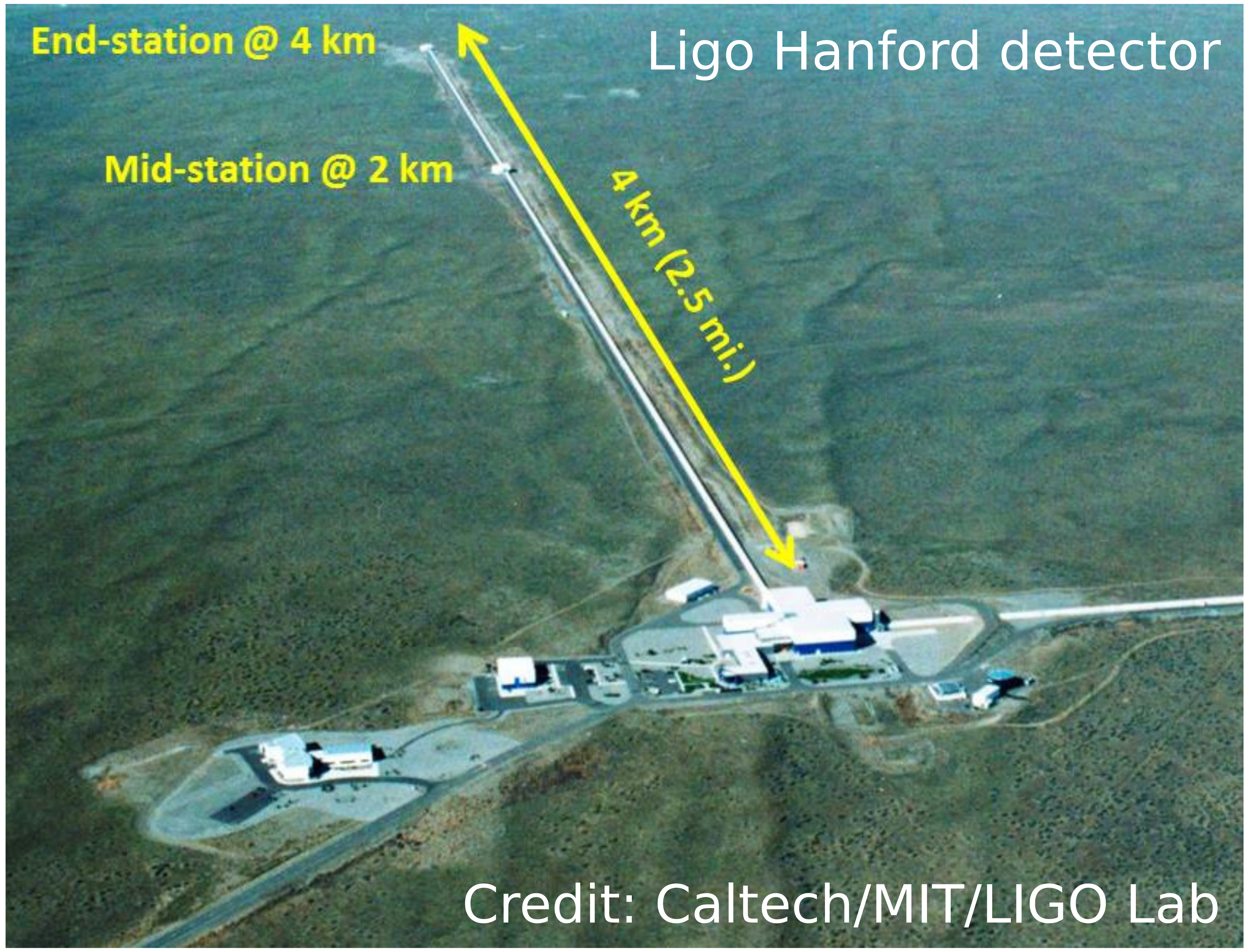


$$\Delta L = 4 \times 10^{-17}\ [\mathrm{m}]$$

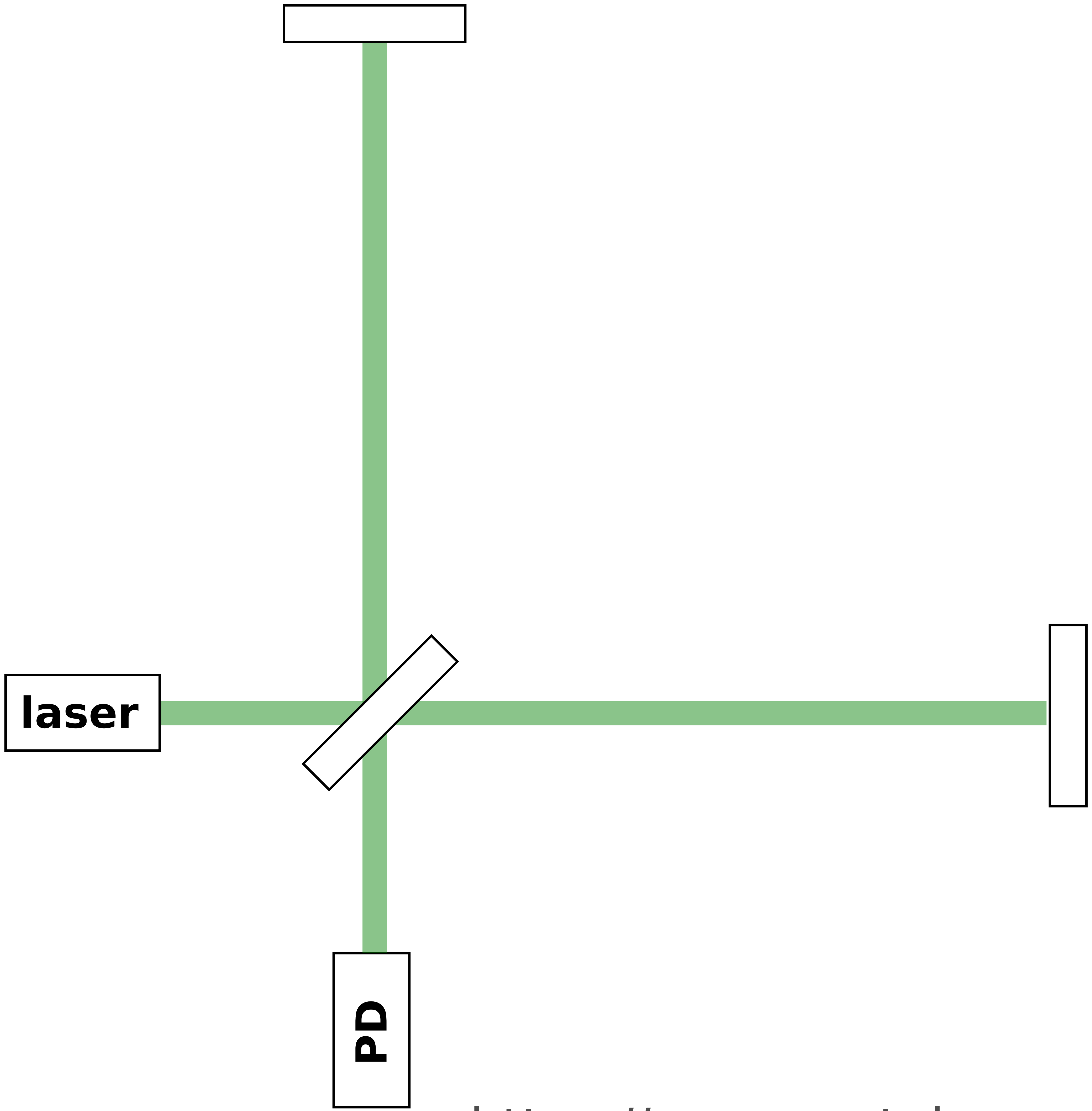
$\text{Human hair: } \sim 10^{-4}\ [\mathrm{m}]$

$\text{Visible light: } \sim 5 \times 10^{-7}\ [\mathrm{m}]$

$\text{Bohr radius: } \sim 5 \times 10^{-11}\ [\mathrm{m}]$

$\text{Proton radius: } \sim 10^{-15}\ [\mathrm{m}]$

Interferometer basics



https://www.youtube.com/watch?v=tQ_teIUb3tE

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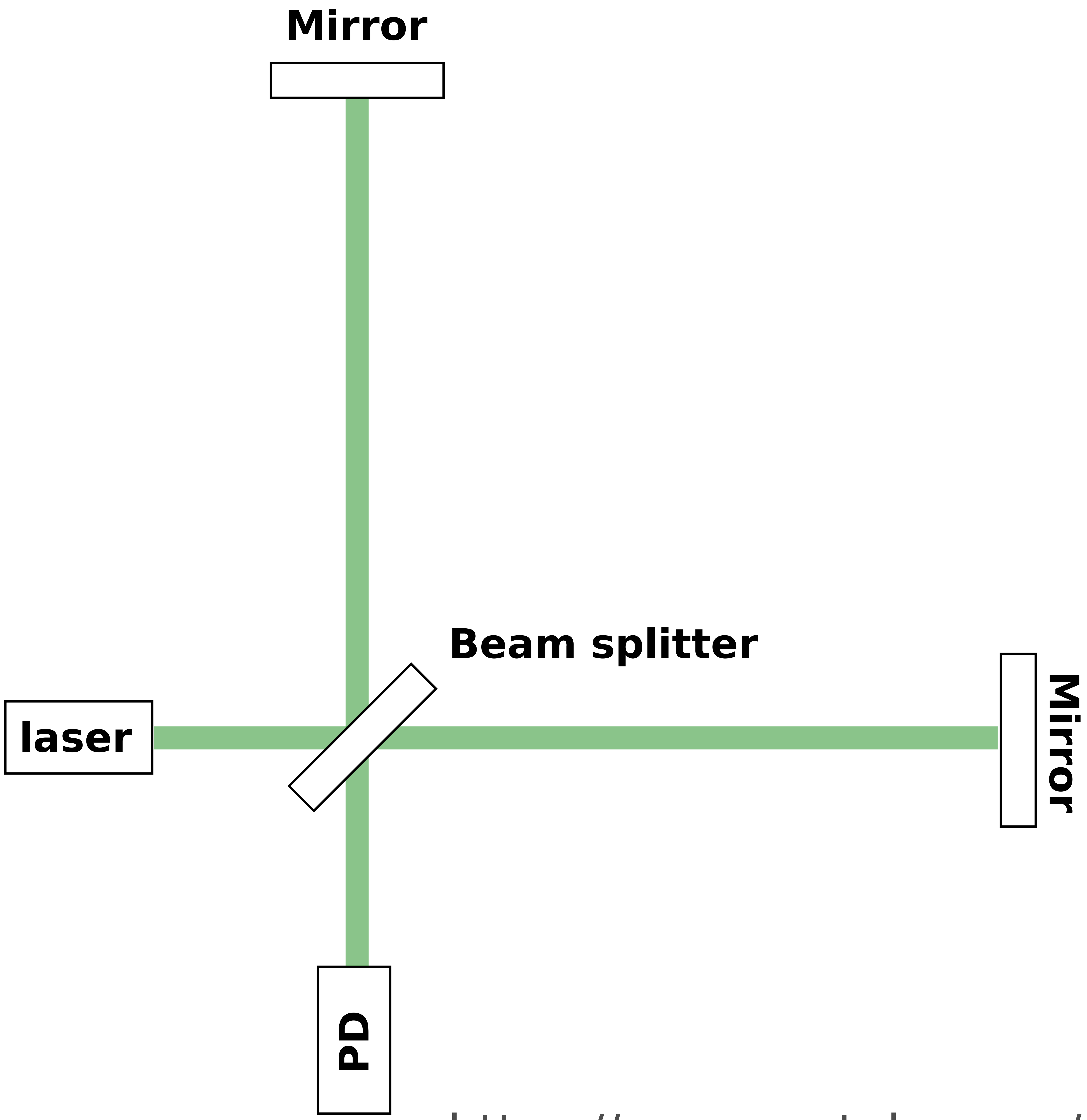

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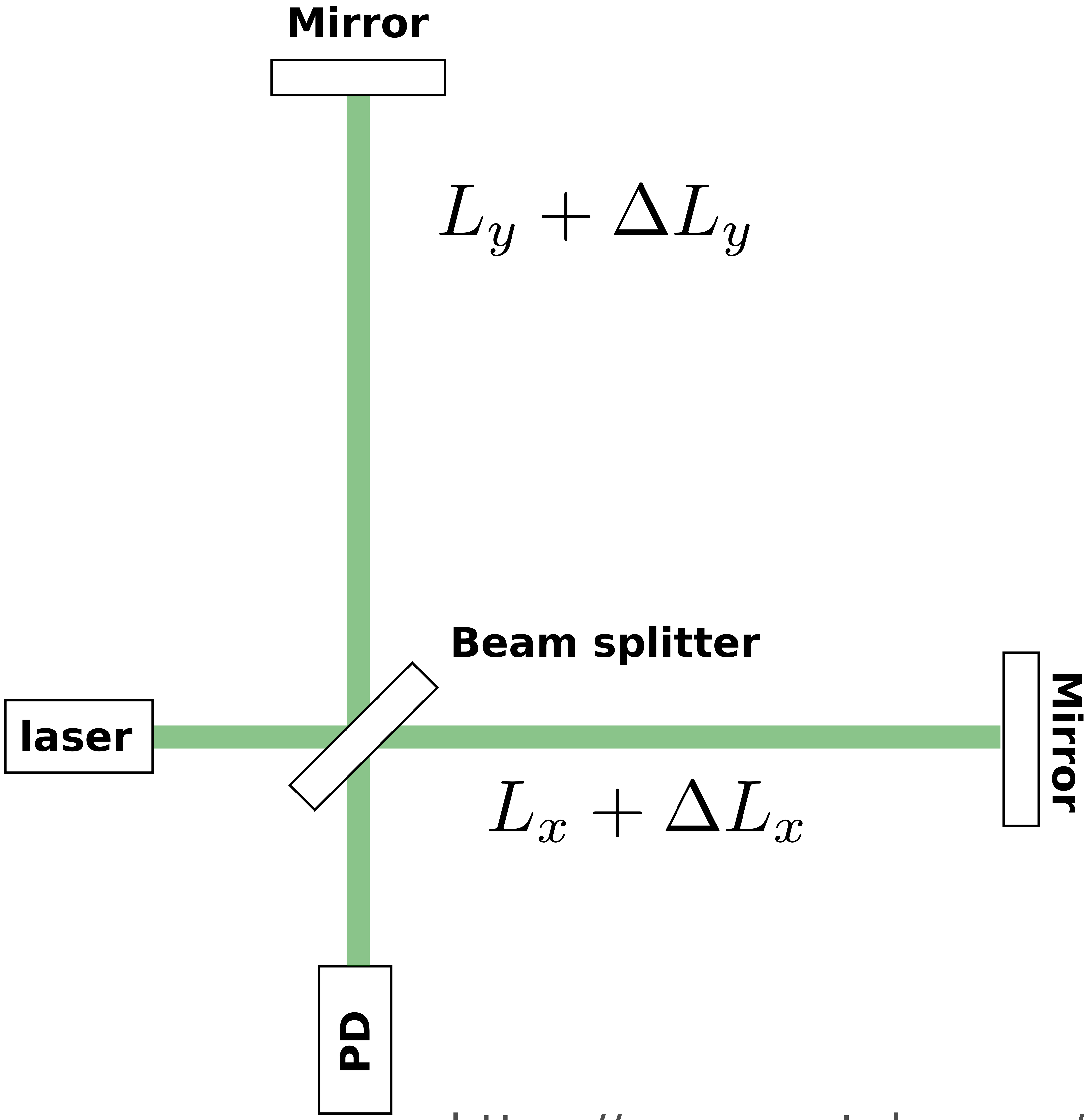

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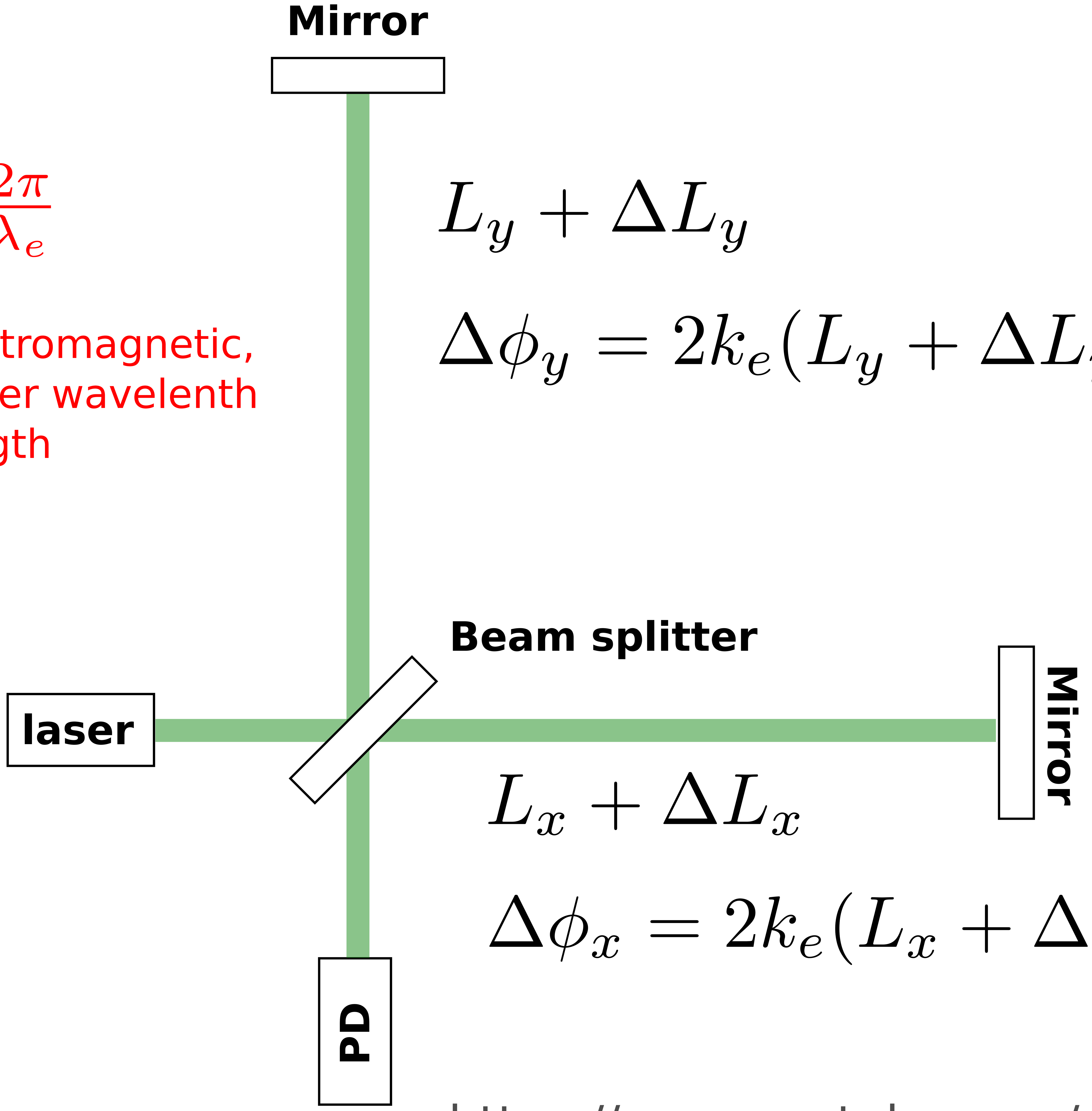
Interferometer basics

$$k_e = \frac{2\pi}{\lambda_e}$$

"e" stands for electromagnetic,
to differentiate laser wavelenth
from GW wavelength



$$L_y + \Delta L_y$$

$$\Delta\phi_y = 2k_e(L_y + \Delta L_y)$$

$$L_x + \Delta L_x$$

$$\Delta\phi_x = 2k_e(L_x + \Delta L_x)$$

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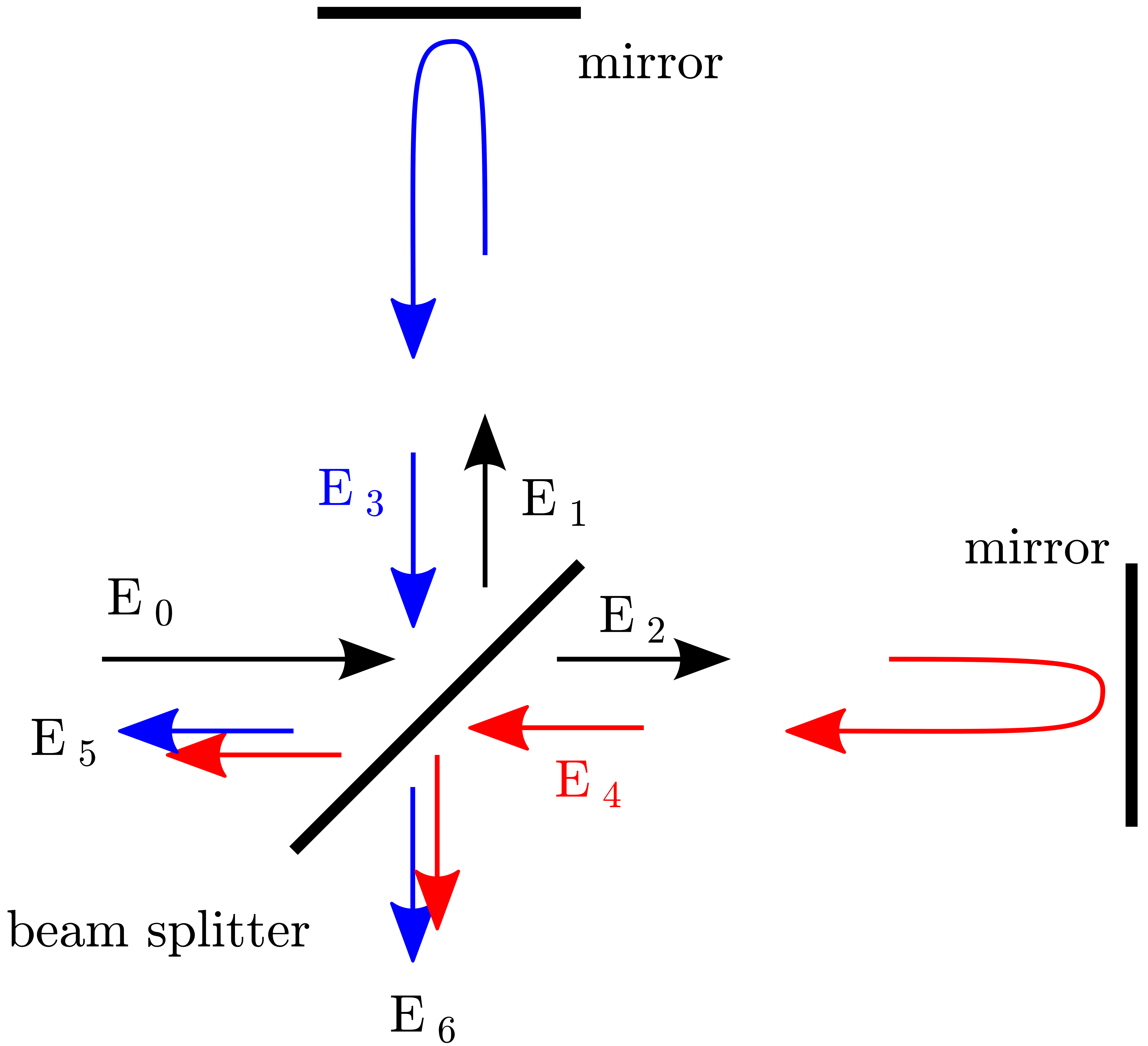

Bond et al. (2016), Living Reviews in Relativity
volume 19, Article number: 3 (2017)

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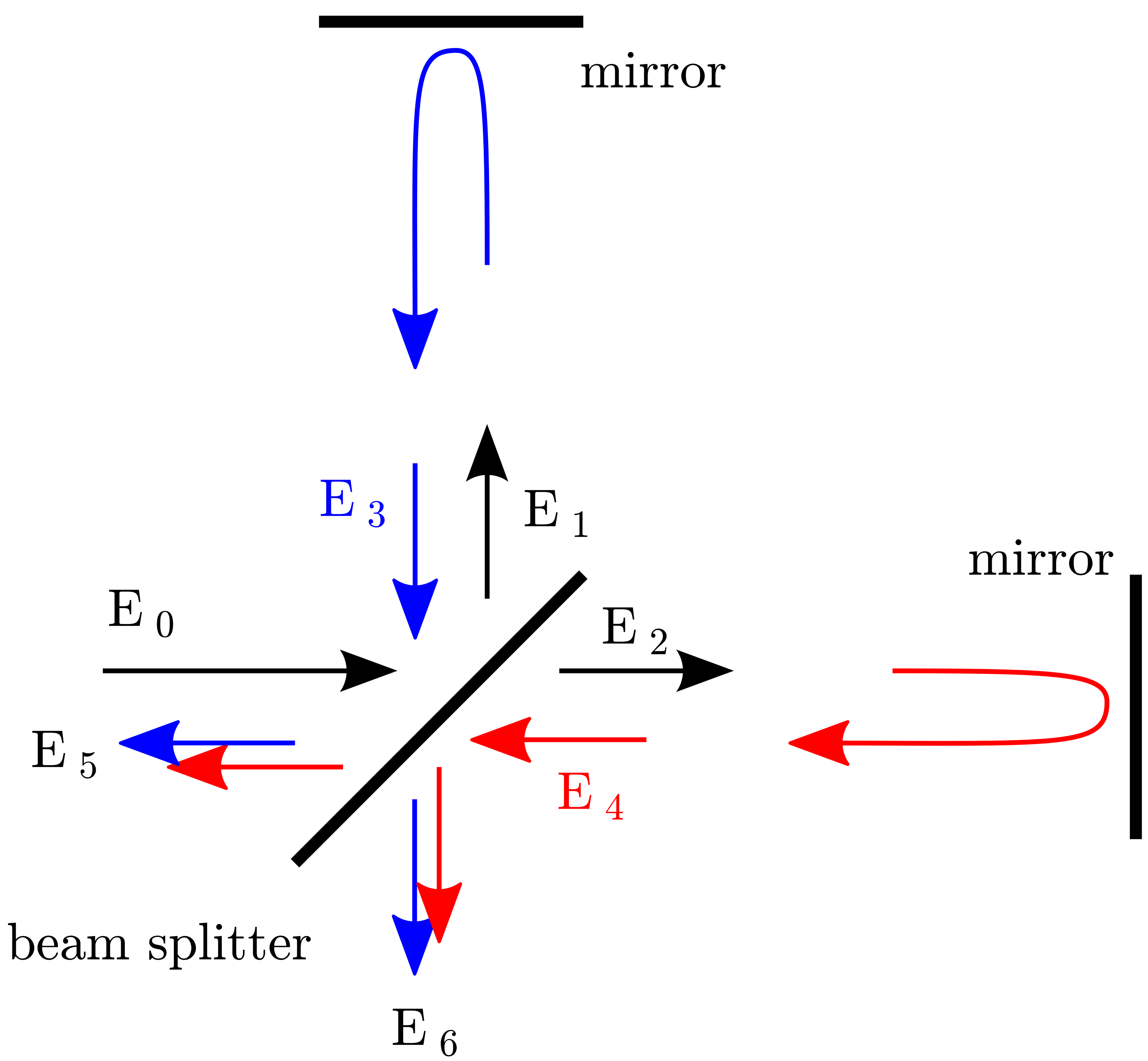

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Beam splitter also induces a phase shift. A perfect beam splitter that separates light into two beams of equal intensity gives:

$$E_1 = \frac{1}{\sqrt{2}} E_0 e^{i\phi_{r1}}, \; E_2 = \frac{1}{\sqrt{2}} E_0 e^{i\phi_t}$$

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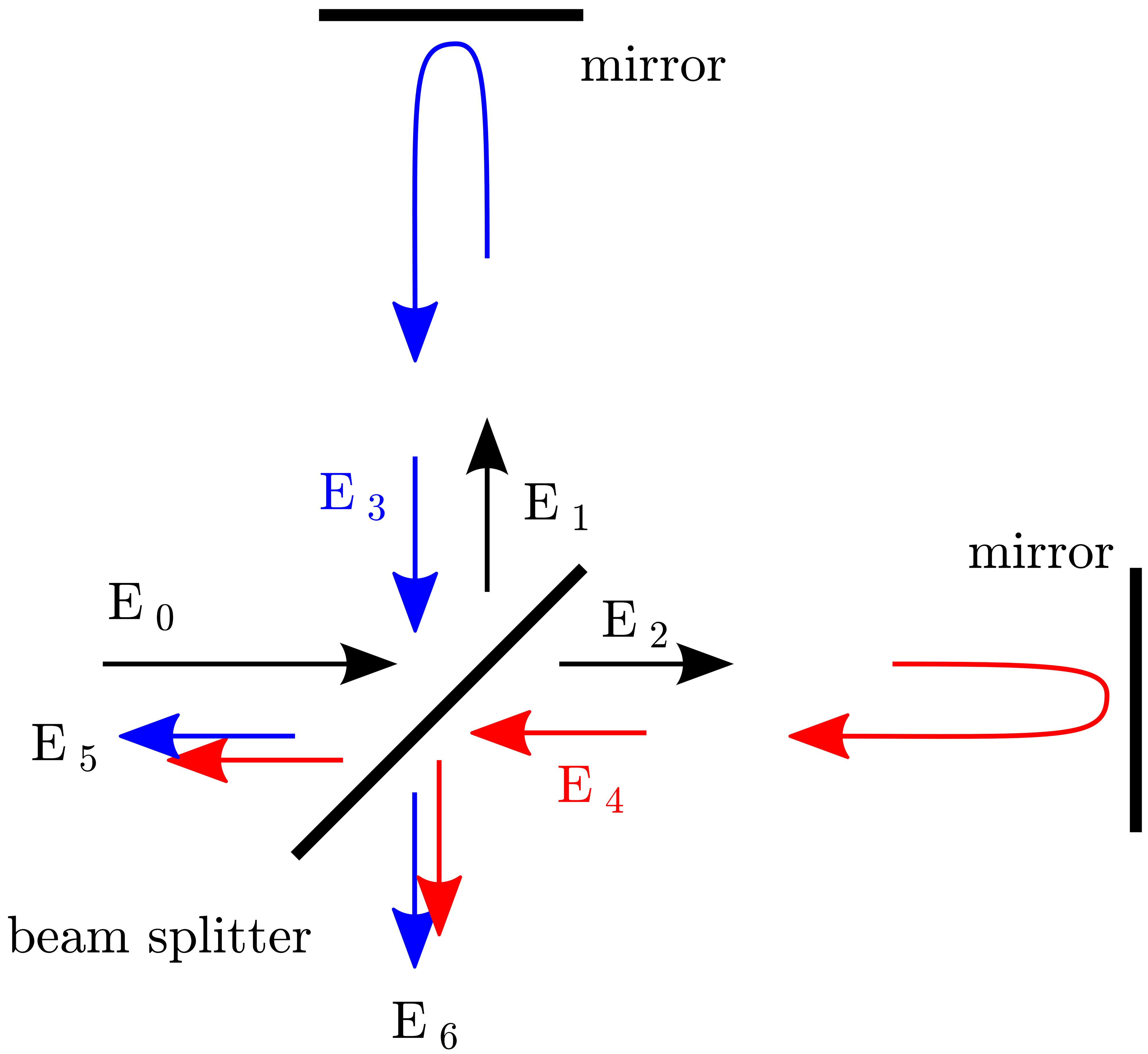

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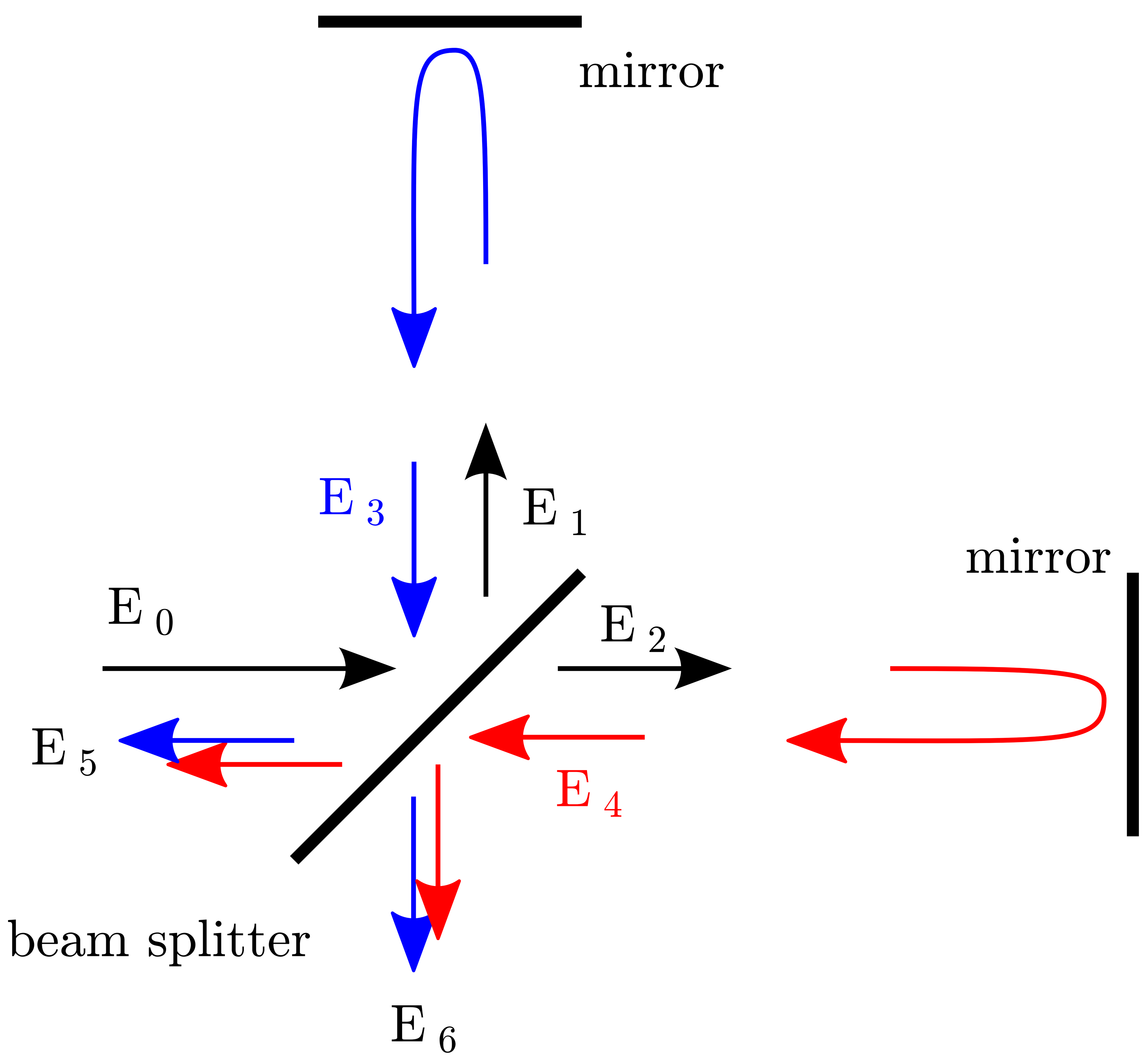

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Similarly, as the beams are bounced from the mirror and return to the beam splitter they are combined after receiving an additional phase shift from the splitter.

$$E_5 = \frac{1}{\sqrt{2}} \left(E_3 e^{i\phi_{r1}} + E_4 e^{i\phi_t} \right)$$

$$E_6 = \frac{1}{\sqrt{2}} \left(E_3 e^{i\phi_t} + E_4 e^{i\phi_{r2}} \right)$$

Interferometer basics

Combining all this gives the amplitude of the wave being sent back to the laser and that being sent to the photodetector

$$E_5 = E_0 e^{i\alpha_+} \cos(\alpha_-),\; E_6 = E_0 e^{i\beta_+} \cos(\beta_-),$$

where:

$$\alpha_- = \phi_{r1} - \phi_t + \tfrac{1}{2}(\Delta\phi_y - \Delta\phi_x)$$

$$\beta_- = \tfrac{1}{2}(\phi_{r1} - \phi_{r2} + \Delta\phi_y - \Delta\phi_x)$$

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Energy conservation connects the intensity going back to the laser to that going to the photodetector.

$$\cos^2\alpha_- + \cos^2\beta_- = 1$$

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Variation at photodetector only sensitive to difference in phase shift

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Combining all this gives the amplitude of the wave being sent back to the laser and that being sent to the photodetector

Exercise 1

- Derive these expressions

- Using conservation of energy determine a relation beween the phase shifts induced by reflection and transmission

where:

Energy ... at going to the photodetector.

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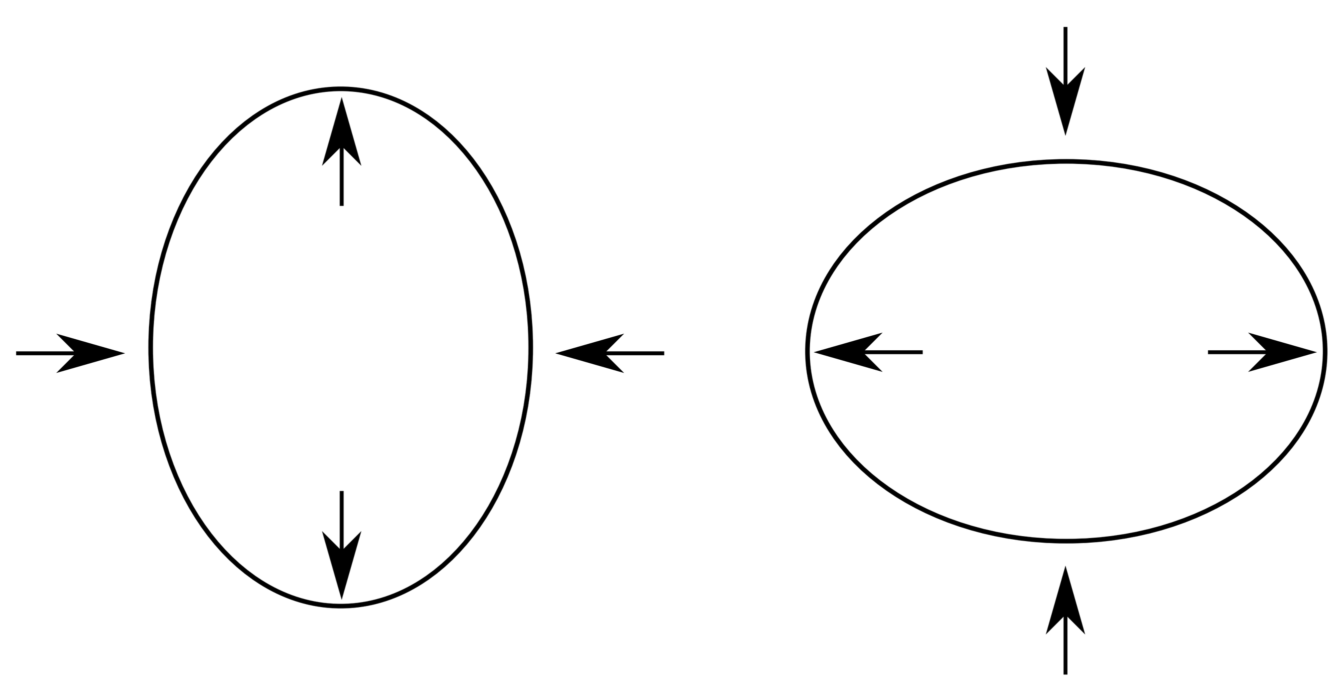
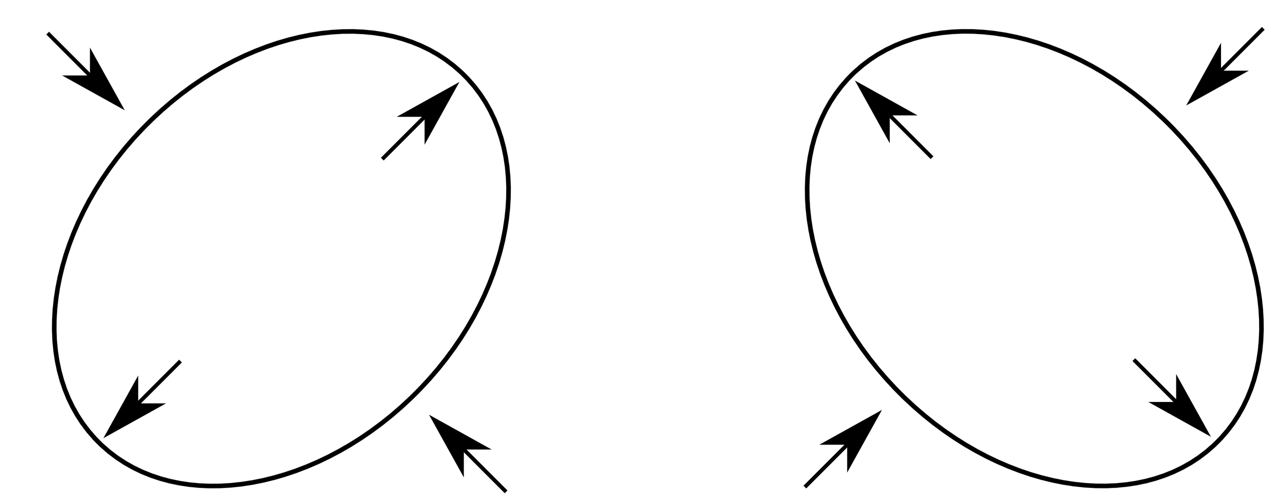
Impact of a GW on interferometer

For a detector with perpendicular arms, arm lengths are sensitive to one GW polarization.

+ polarization



x polarization



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Taking an interferometer aligned with a + polarized wave, the change in arm lengths can be derived from the perturbation to the spacetime metric induced by the wave

$$ds^2 = -c^2 dt^2 + (1 + h_+(t,z))dx^2 + (1 - h_+(t,z))dy^2 + dz^2$$

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$$\Delta L_x = \tfrac{1}{2}h(t)L_x,\ \Delta L_y = -\tfrac{1}{2}h(t)L_y$$

Impact of a GW on interferometer

This is a tiny shift in phase at the photodetector:

$$\Delta\phi_x - \Delta\phi_y = hk_eL \sim \frac{10^{-10}}{2\pi}\left(\frac{L}{4\ \mathrm{km}}\right)\left(\frac{\lambda_e}{1000\ \mathrm{nm}}\right)^{-1}$$

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Ideally, for a target GW frequency we would use arms with lengths equal to a quarter of the GW wavelength. Smaller arms will result in a smaller phase shift, longer arms will give negating contributions to the phase shift as the deformation shifts between arms.

$$L_{\mathrm{ideal}} = 750\ [\mathrm{km}]\left(\frac{f}{100\ \mathrm{Hz}}\right)^{-1}$$

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Not really ideal to build such a large detector...

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Exercise 2

Determine the ideal length of an interferometer designed to operate at frequencies of 10^-2 and 10^-8 Hz. Express your results in solar radii and parsecs respectively.

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a qua … phase
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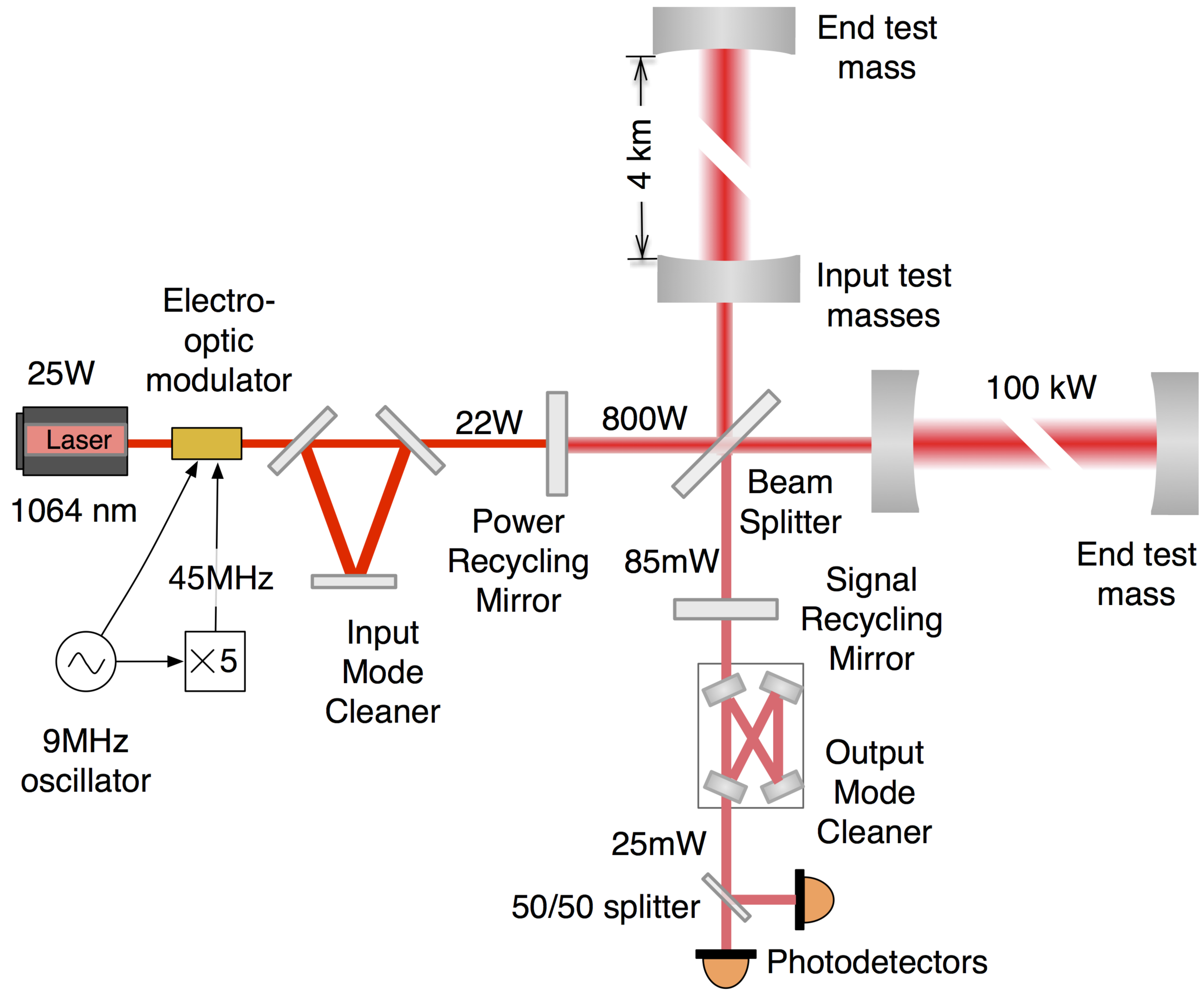
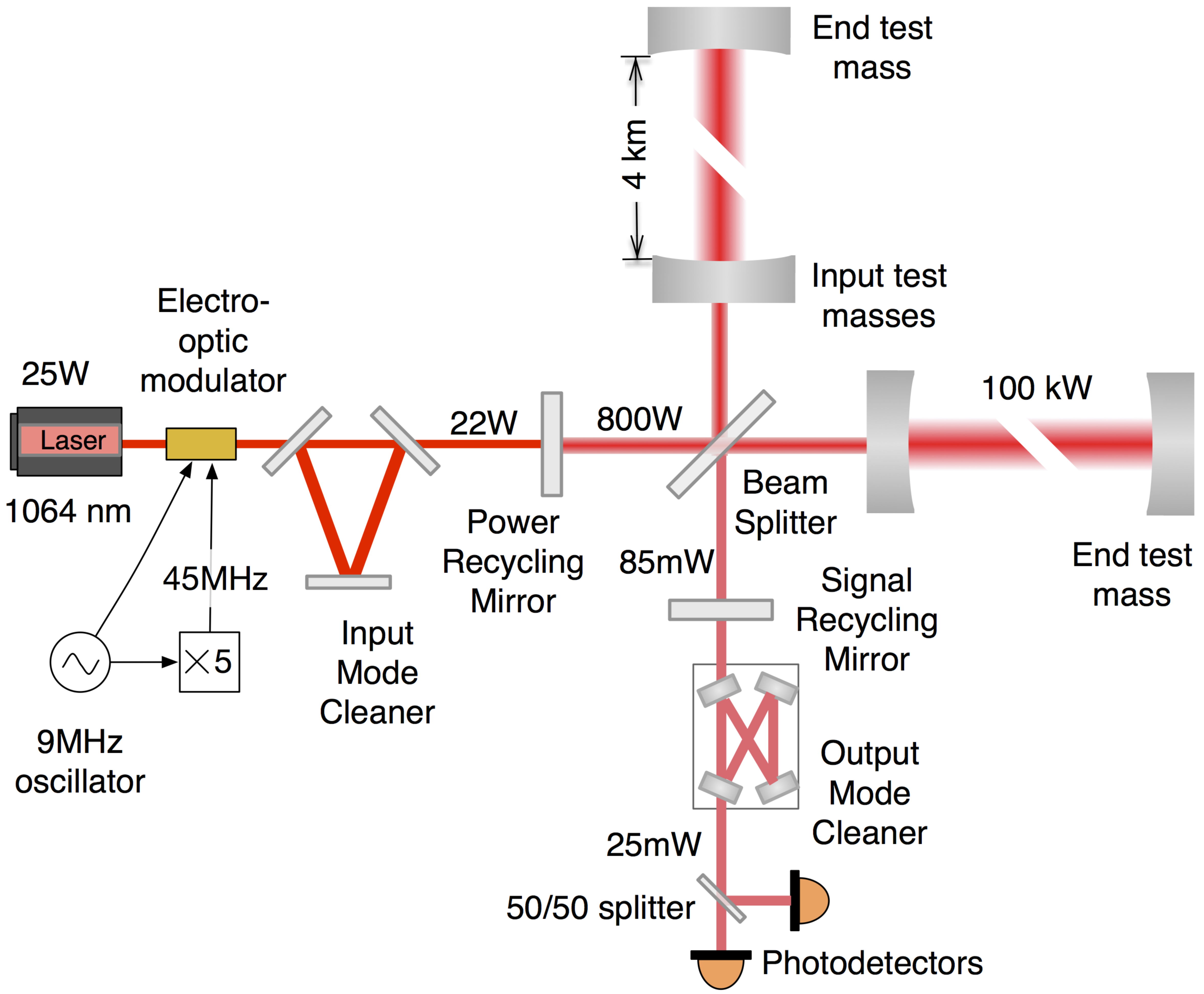
Fabry Perot cavities



Each arm contains Fabry-Perot optical cavities where light is made to bounce ~300 times before exiting back to the beam splitter. This increases the target phase shift we want to measure.

Fabry Perot cavities



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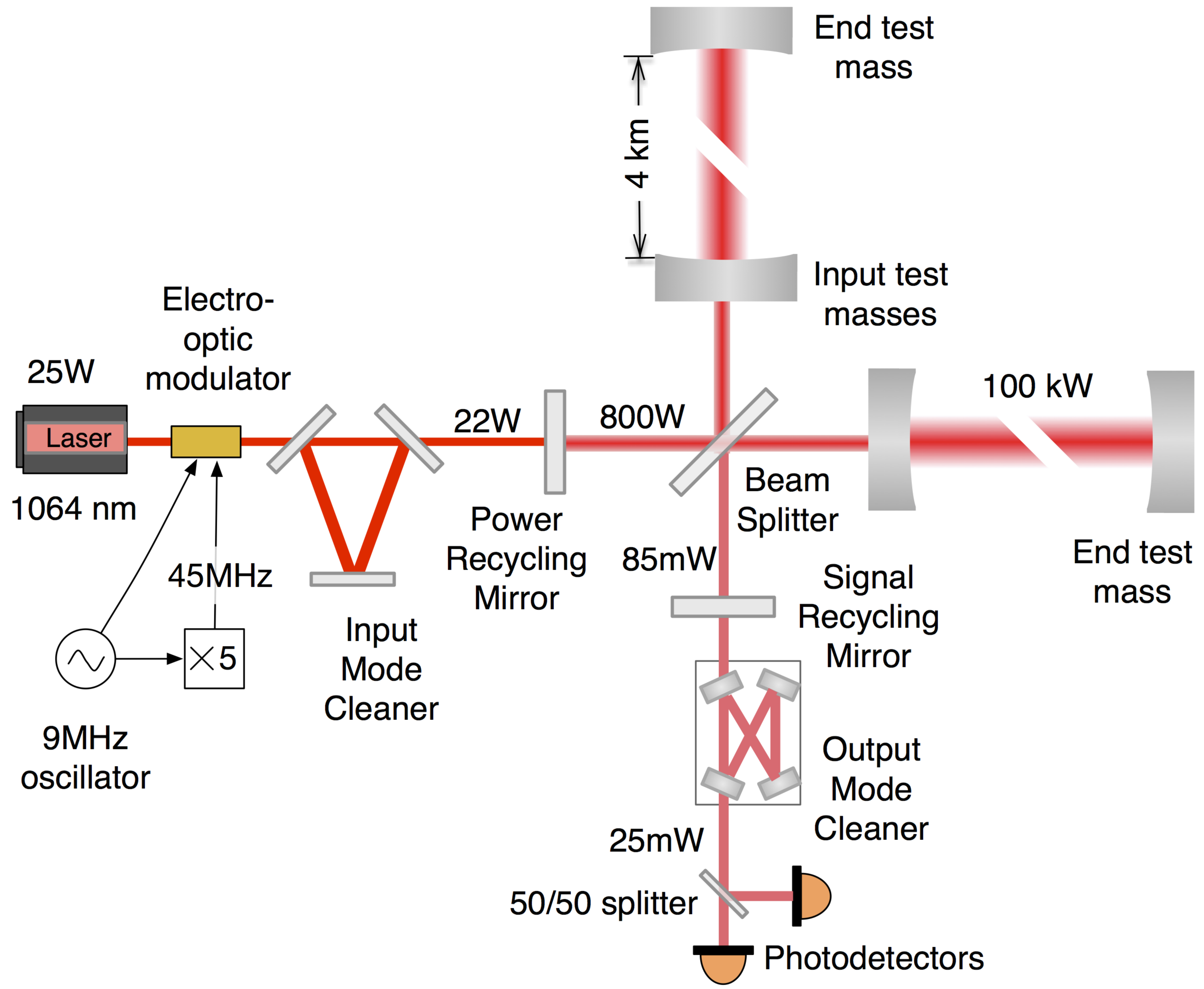

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Number of bounces

Requirement on laser power

This measurement in phase can be understood as a precision on a time measurement. For the specific case of a 100 Hz wave being bounced ~100 times in a 4 kilometer arm, we have that

$$\omega_e \Delta t = \Delta \phi$$

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In its simplest form, we integrate the energy of the laser (with power P) over a fraction of a gravitational wave period. We have an expected number of photons detected in this interval with a Poissonian error,

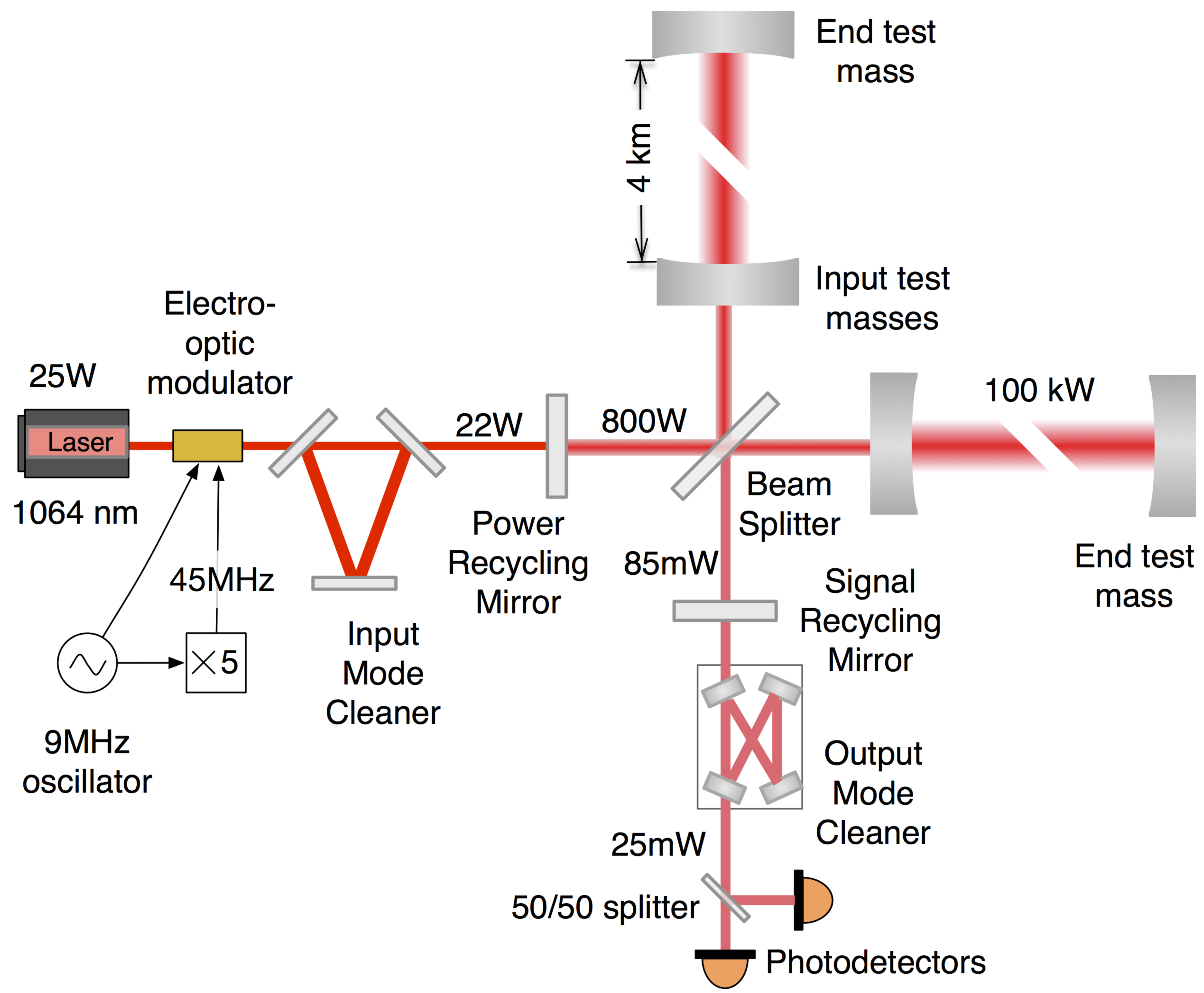
$$E = N_\gamma \hbar\omega_e,\; N_\gamma = \frac{P}{\hbar\omega_e} \times \frac{1}{4f},\; \Delta E \sim \sqrt{N_\gamma}\hbar\omega_e$$

Requirement on laser power

Combining all this we can compute the required laser power needed for the error in phase to match a given GW signal:

$$P \gtrsim \frac{4 f \hbar \omega_e}{(h k_e L B)^2}$$

Plugging the numbers we've seen for LIGO (4 km arm length, ~100 bounces, 1000 nm laser, and a 100 Hz source, 10^-21 strain) we find a power of 10 watts. LIGO's laser is around this, but it uses a trick called power recycling to boost the power inside the interferometer.



Requirement on laser power

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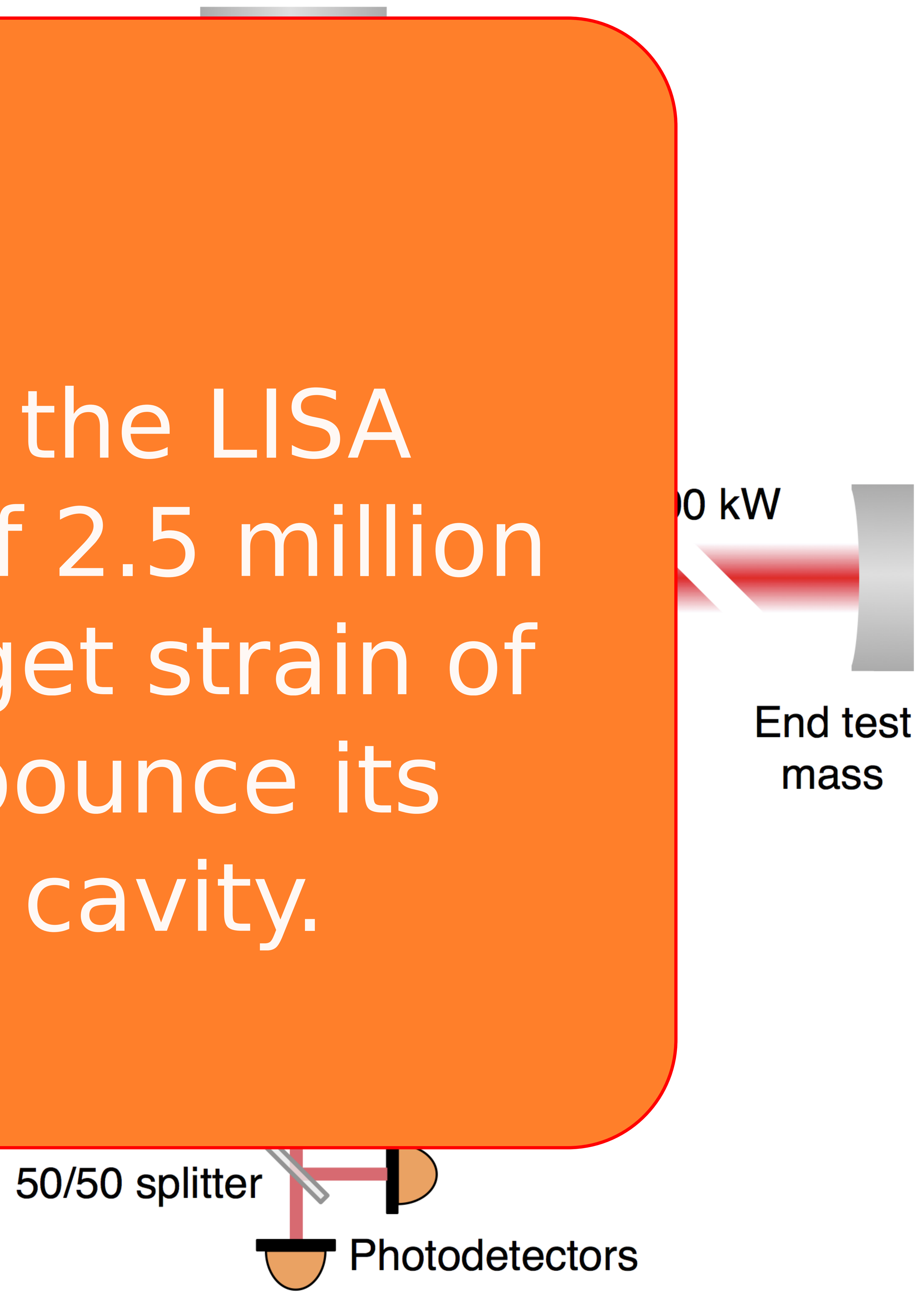
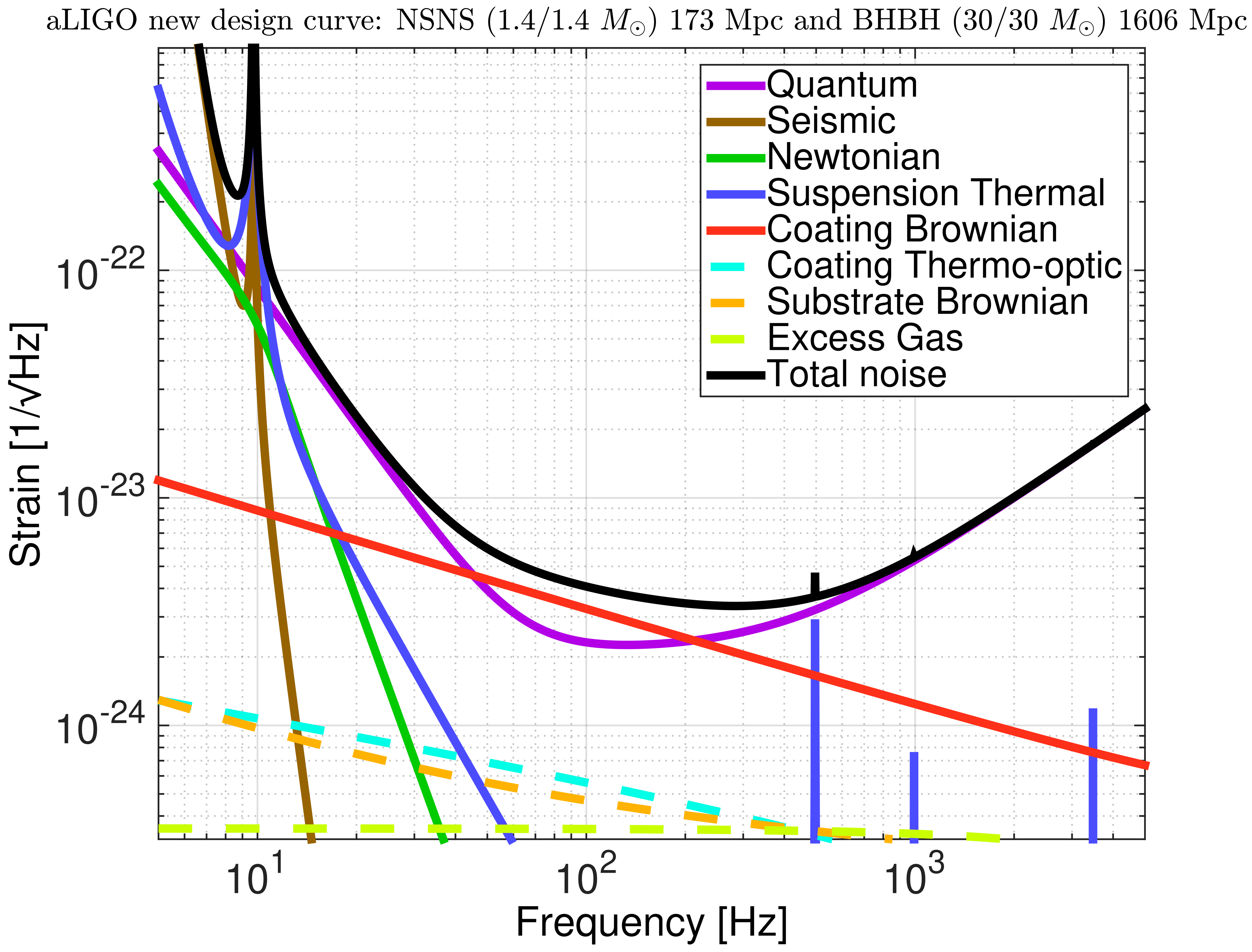
Exercise 3

What is the laser power required by the LISA interferometer? Assume arm lengths of 2.5 million kilometers, a 1000 nm laser and a target strain of 10^{-21}. Note that LISA does not bounce its lasers back and forth in an optical cavity.



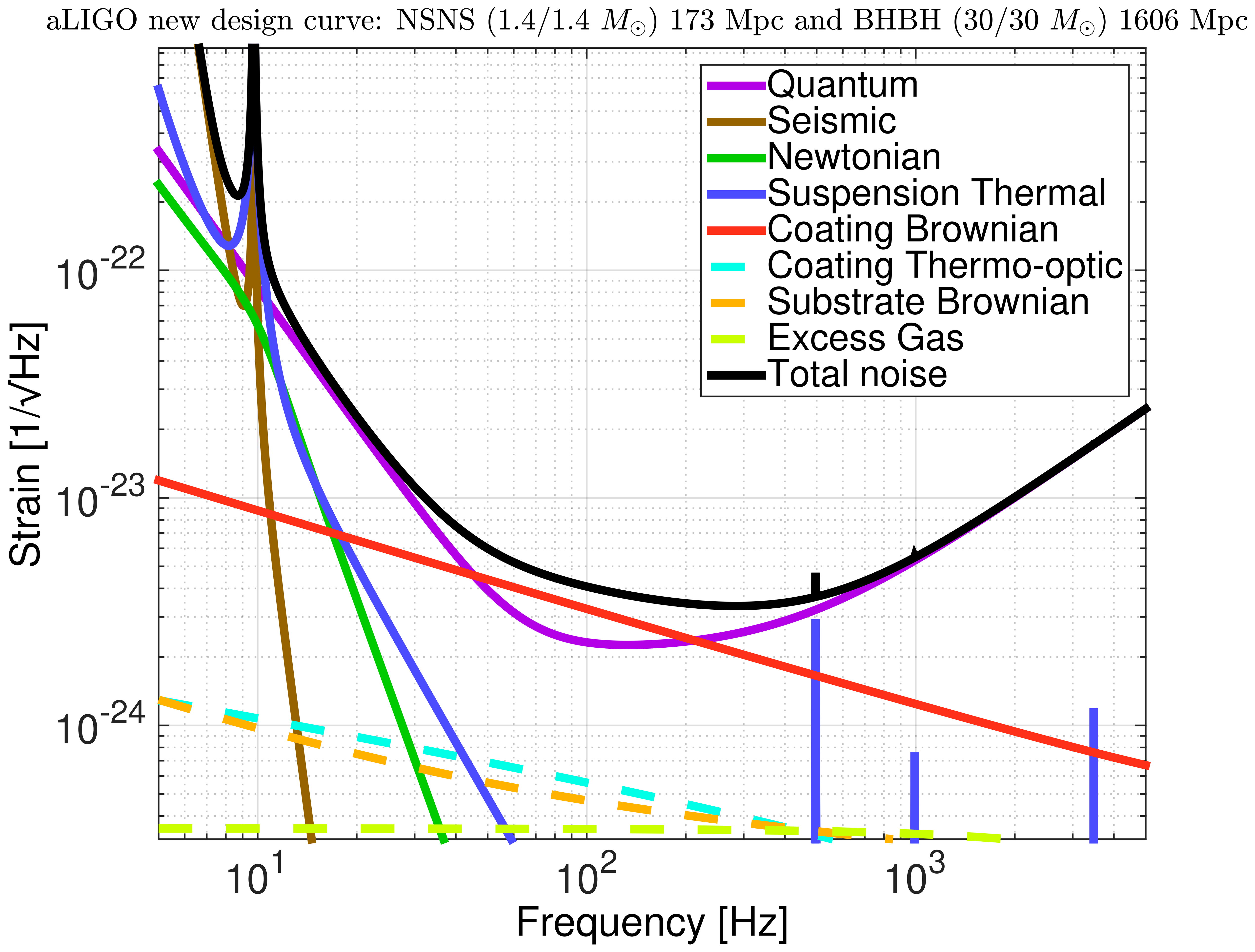
Full noise budget



https://dcc.ligo.org/LIGO-T1800044/public

What we have explored is just one form of noise, quantum shot noise. Quantum noise from the laser light also comes in the form of radiation pressure noise at low frequency.

Full noise budget



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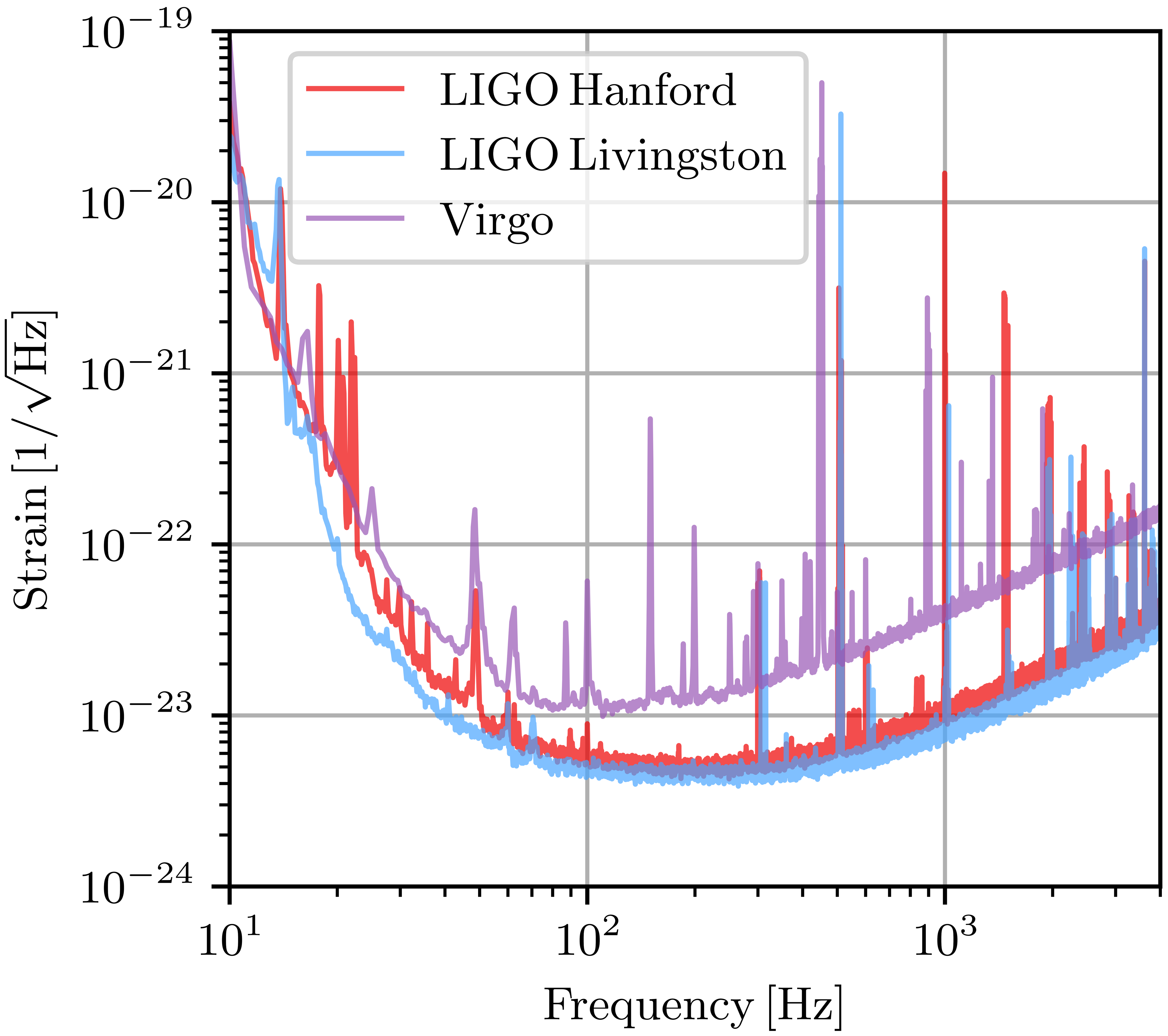
Noise budget below 10 Hz dominated by seismic noise, LIGO requires elaborate seismic isolation systems!

Full noise budget

Actual instrument noise is very complex!



astro-ph: 2010.14527

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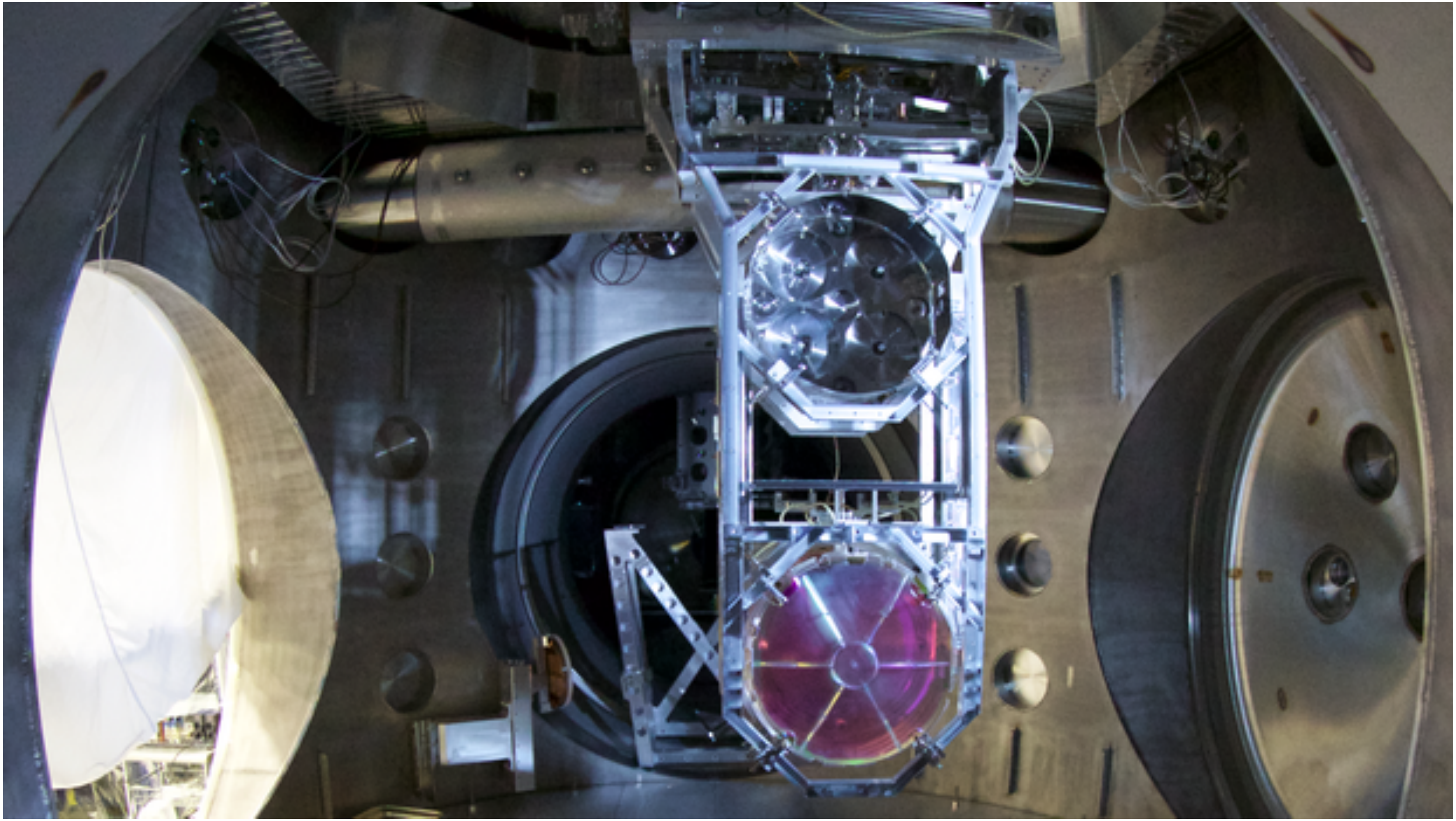
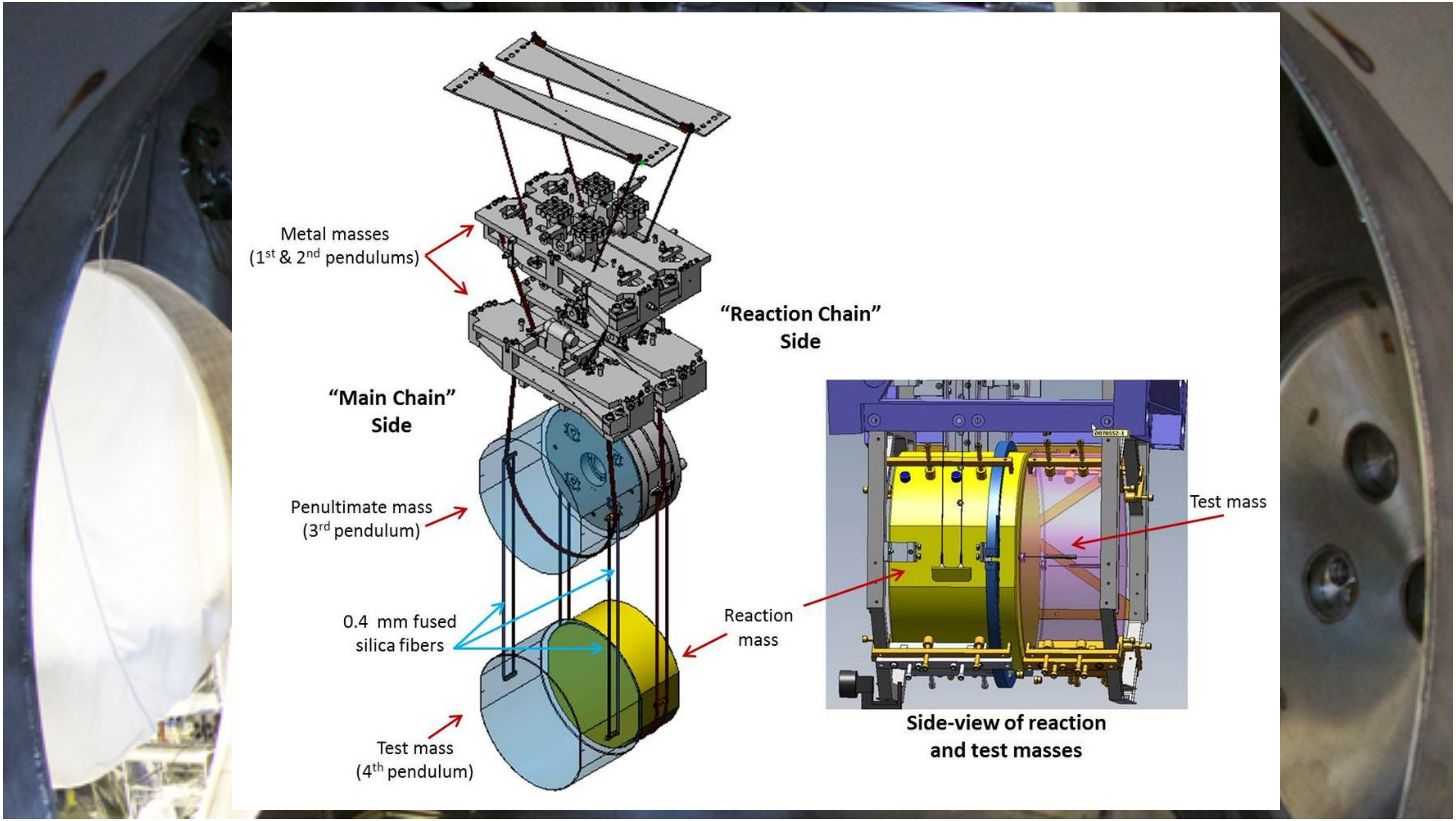
Seismic isolation



Seismic isolation system for one mirror in LIGO

credit: Caltech/MIT/LIGO Lab/Greg Grabeel

Seismic isolation



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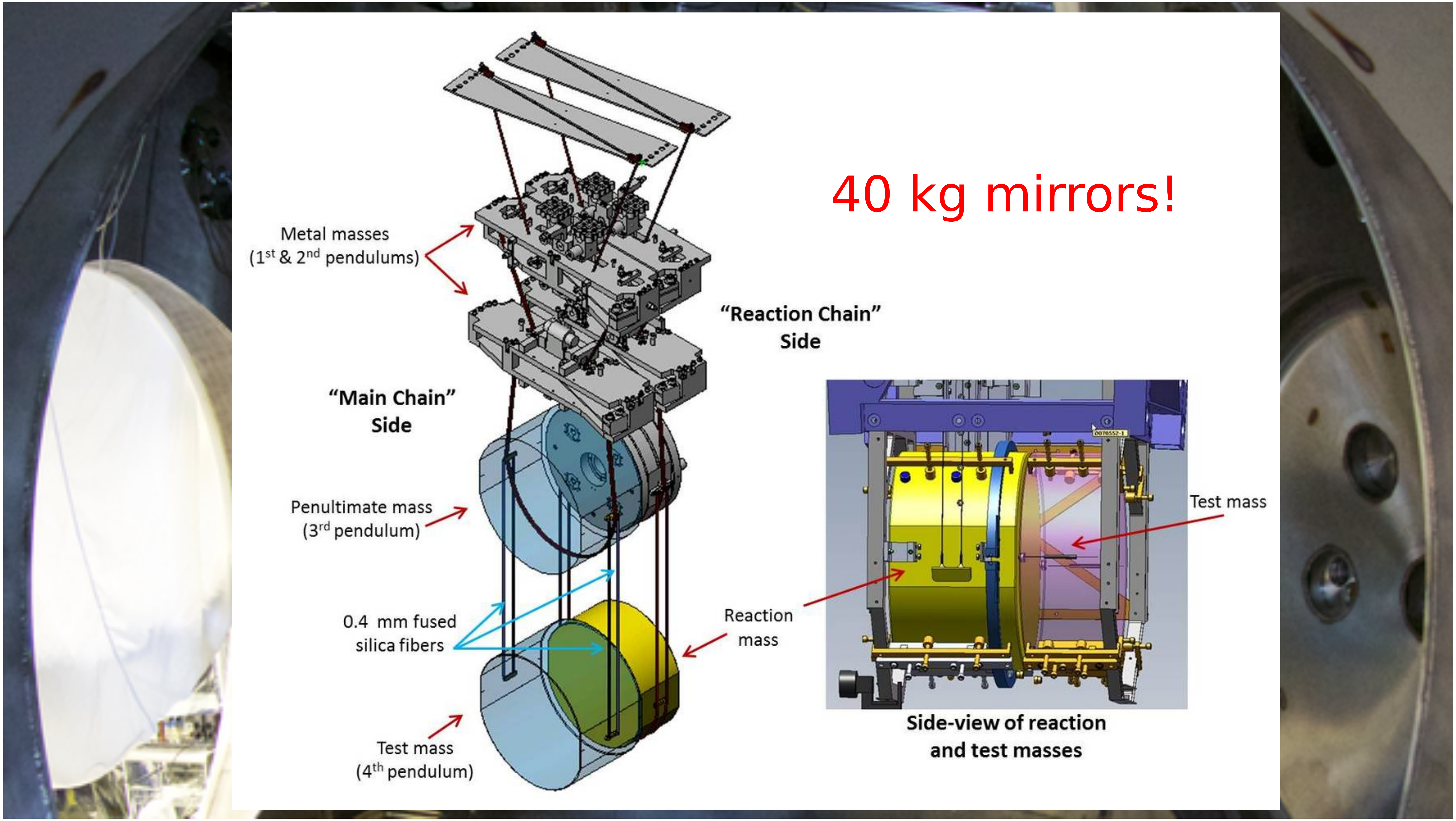

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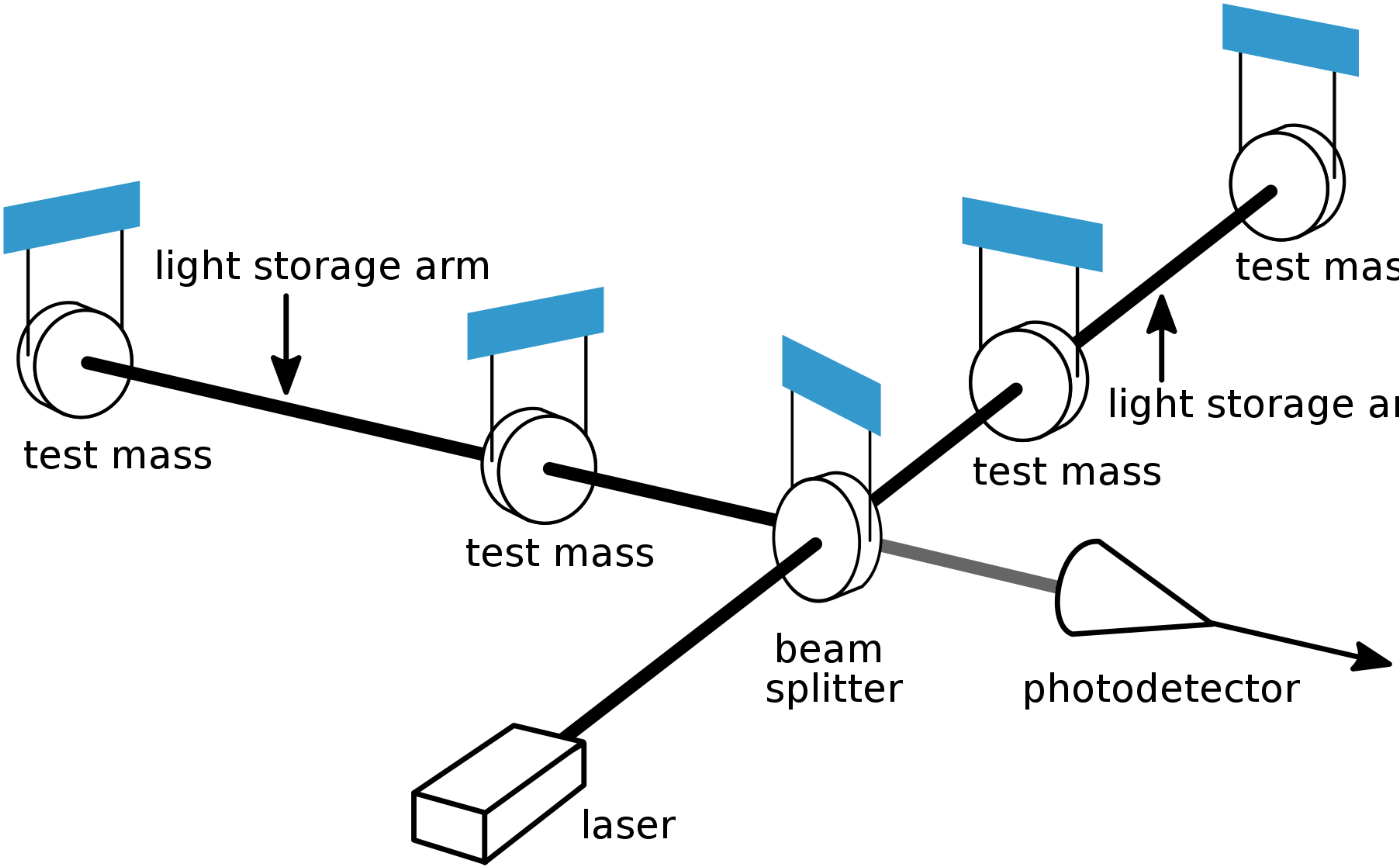

MOBIe user, wikipedia

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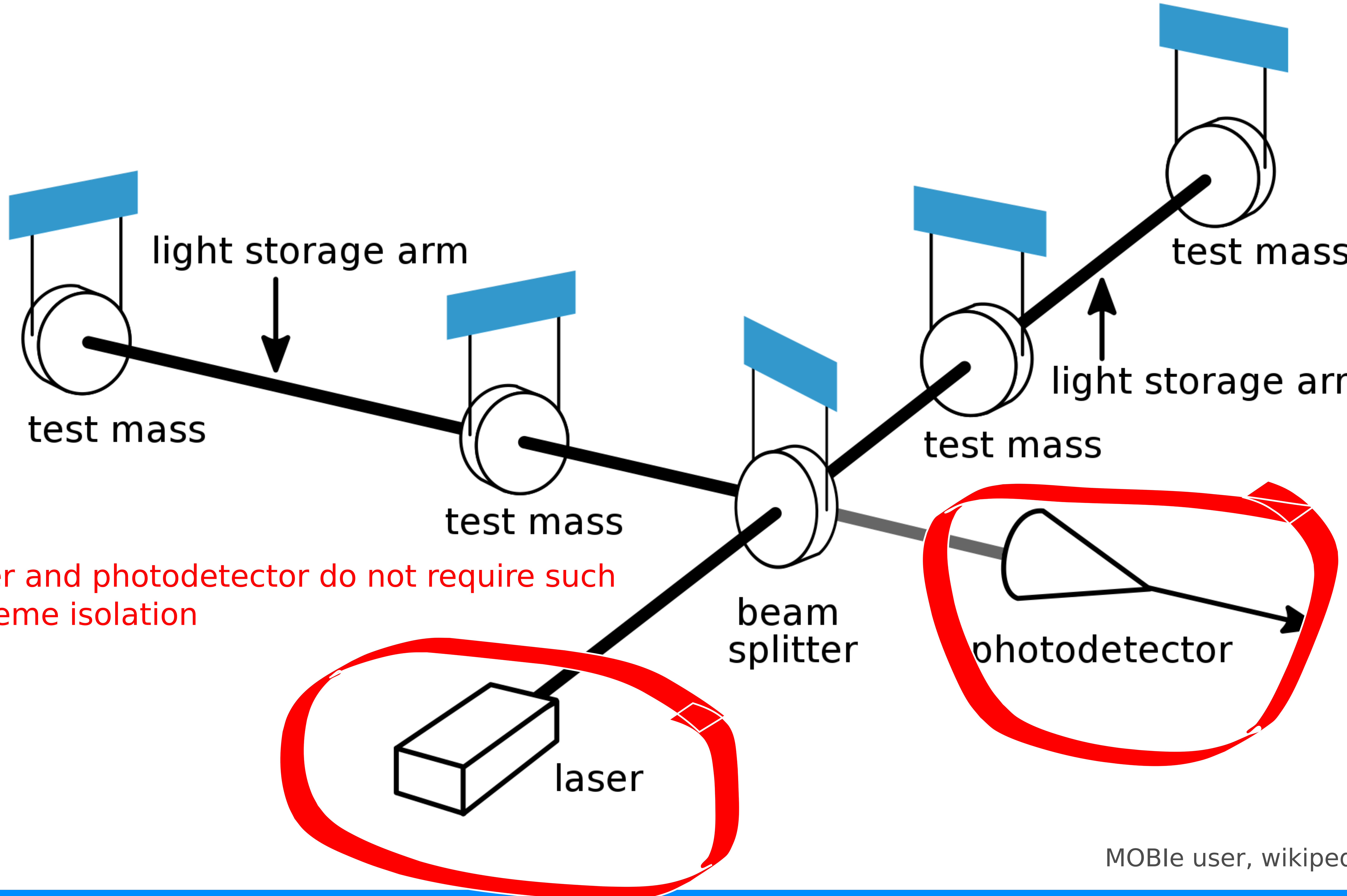
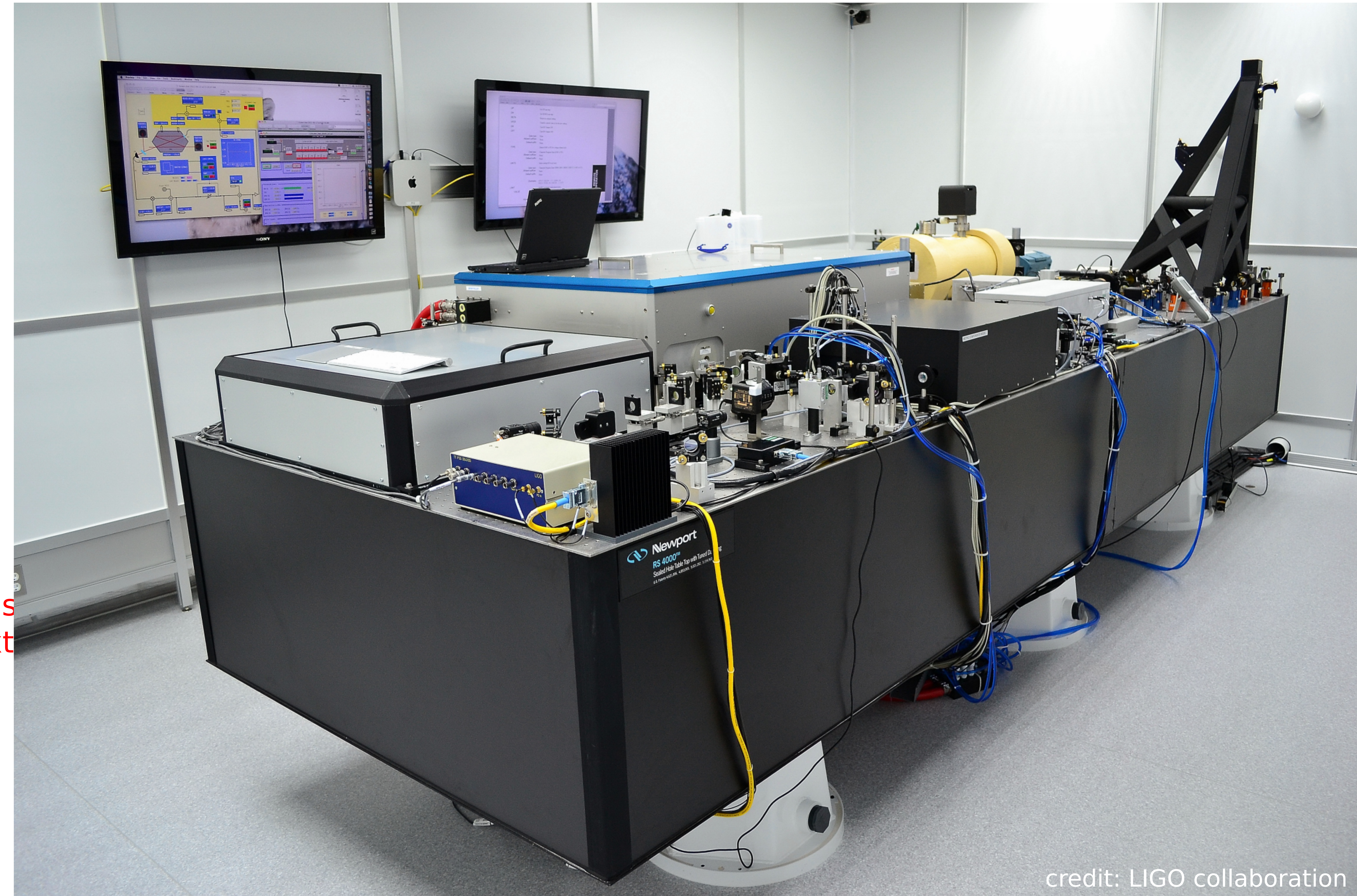


Laser and photodetector do not require such extreme isolation

MOBIe user, wikipedia

Seismic isolation



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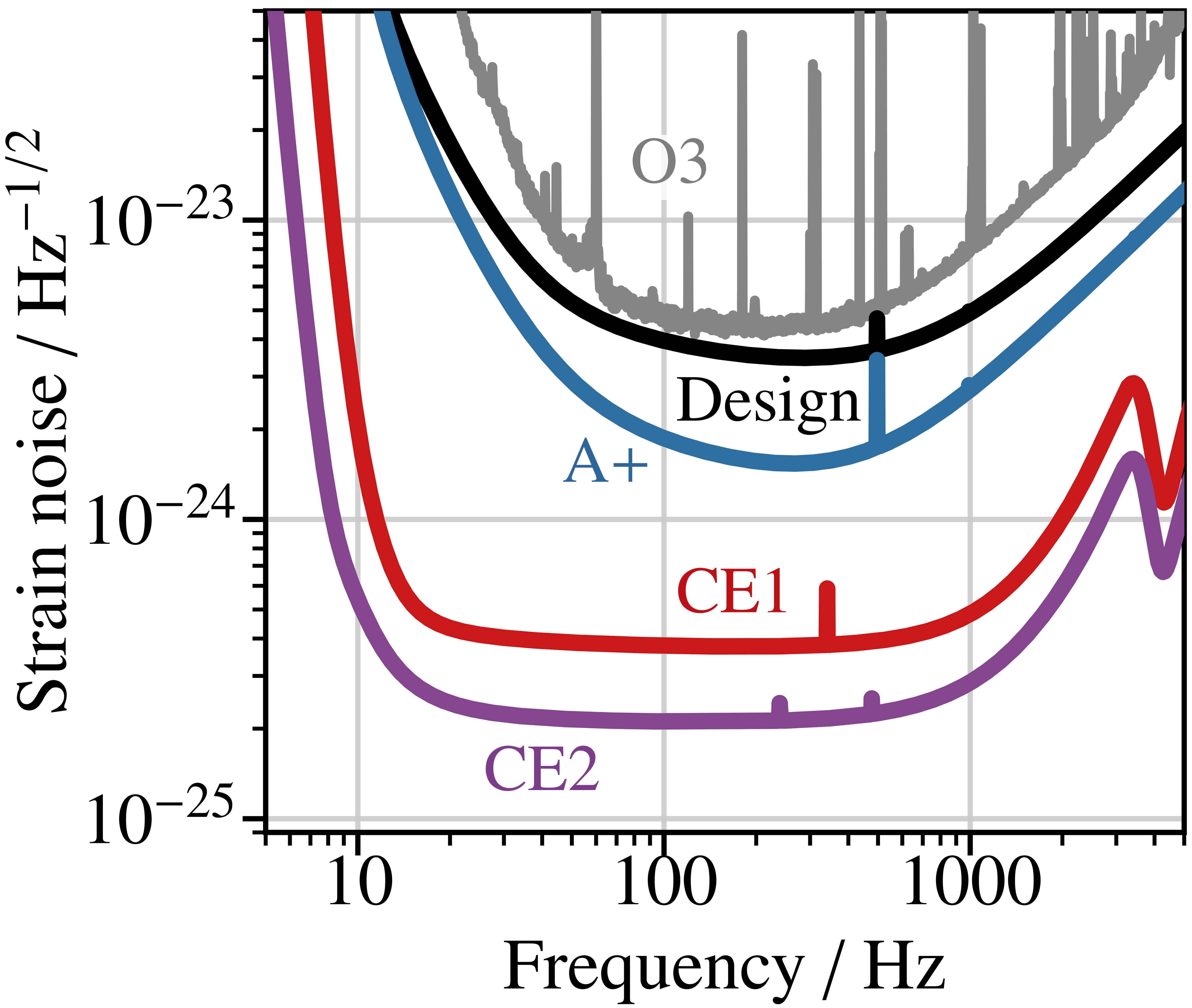
To finish: Noise spectral density



Reitze et al. (2019)

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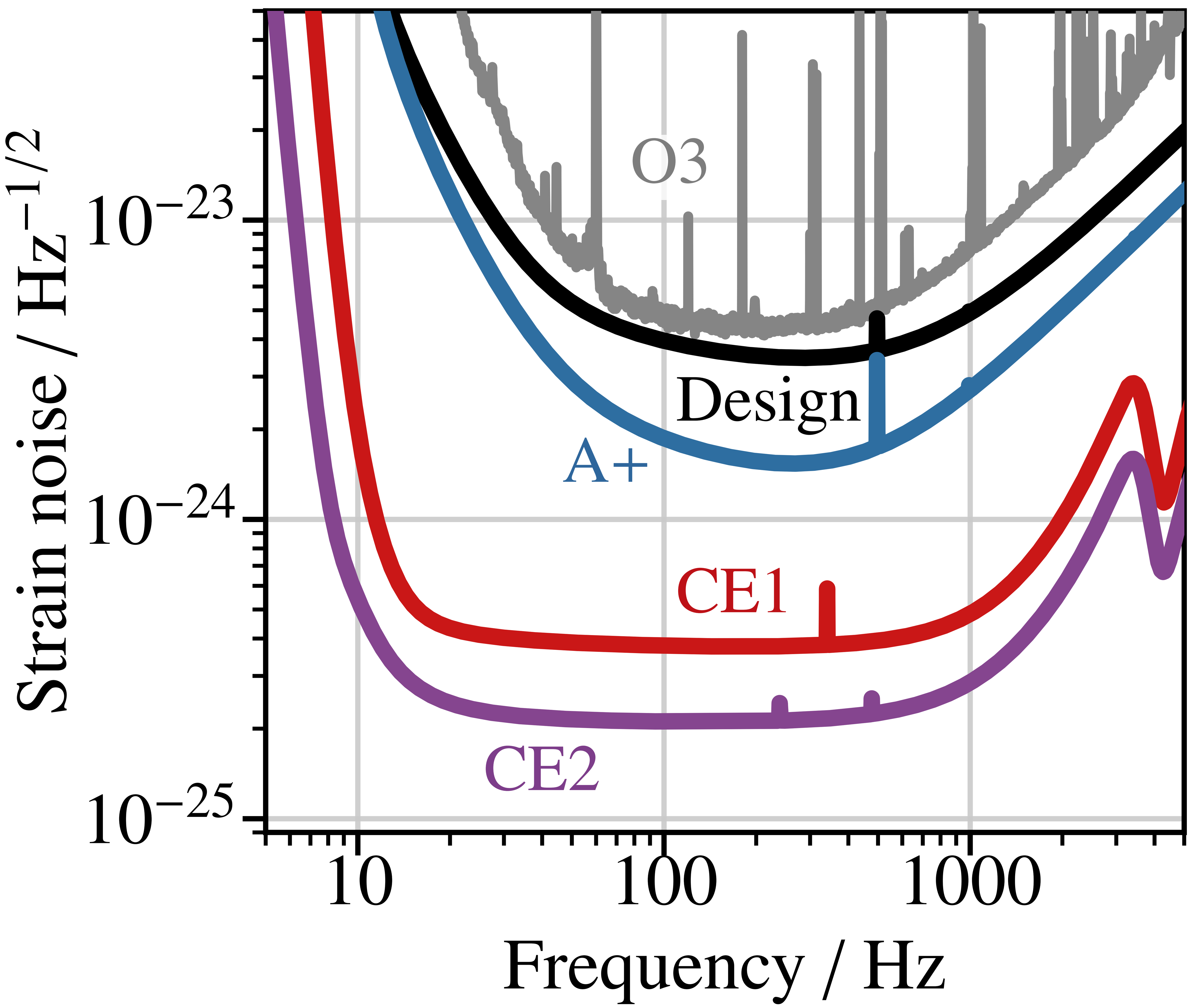

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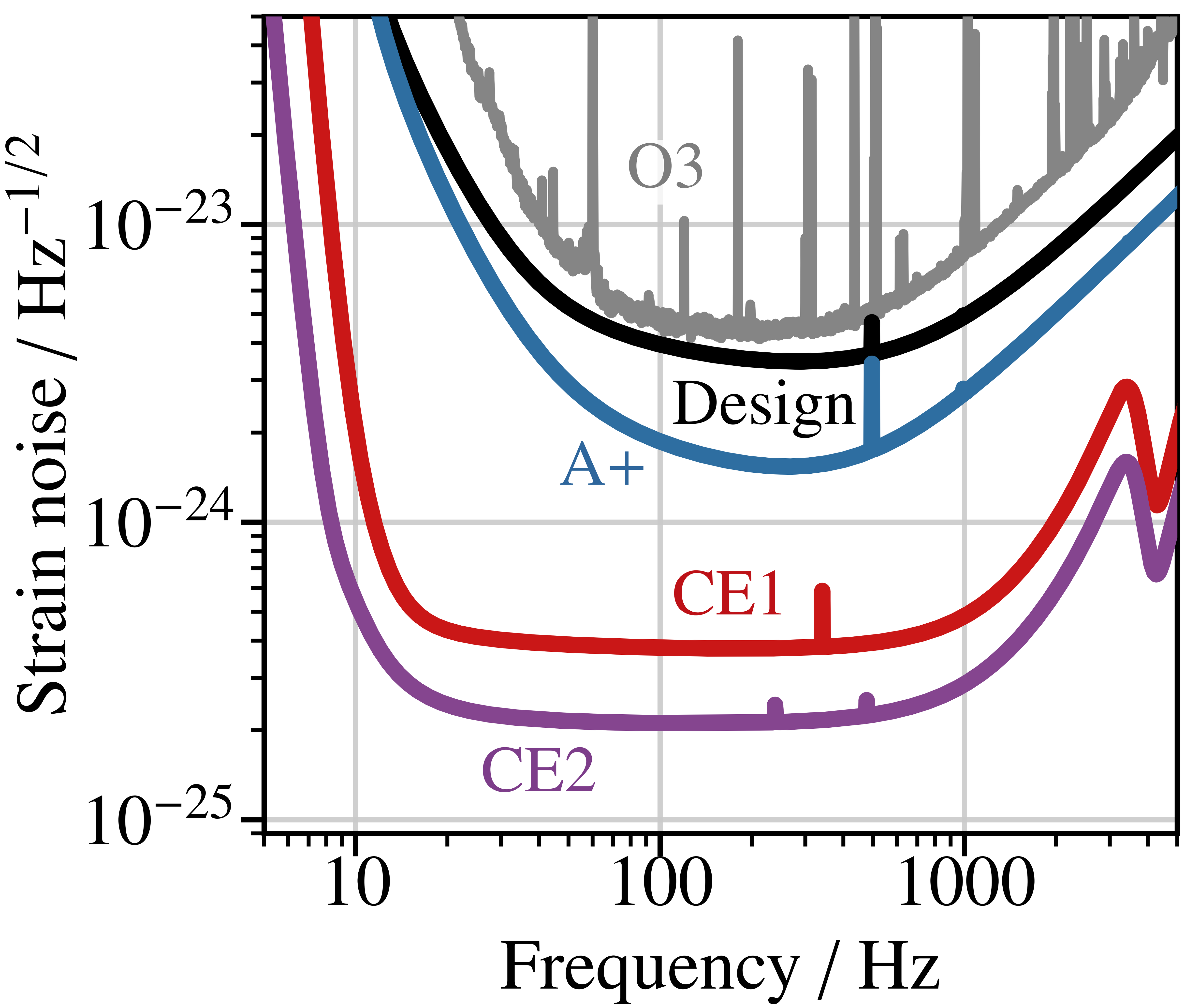

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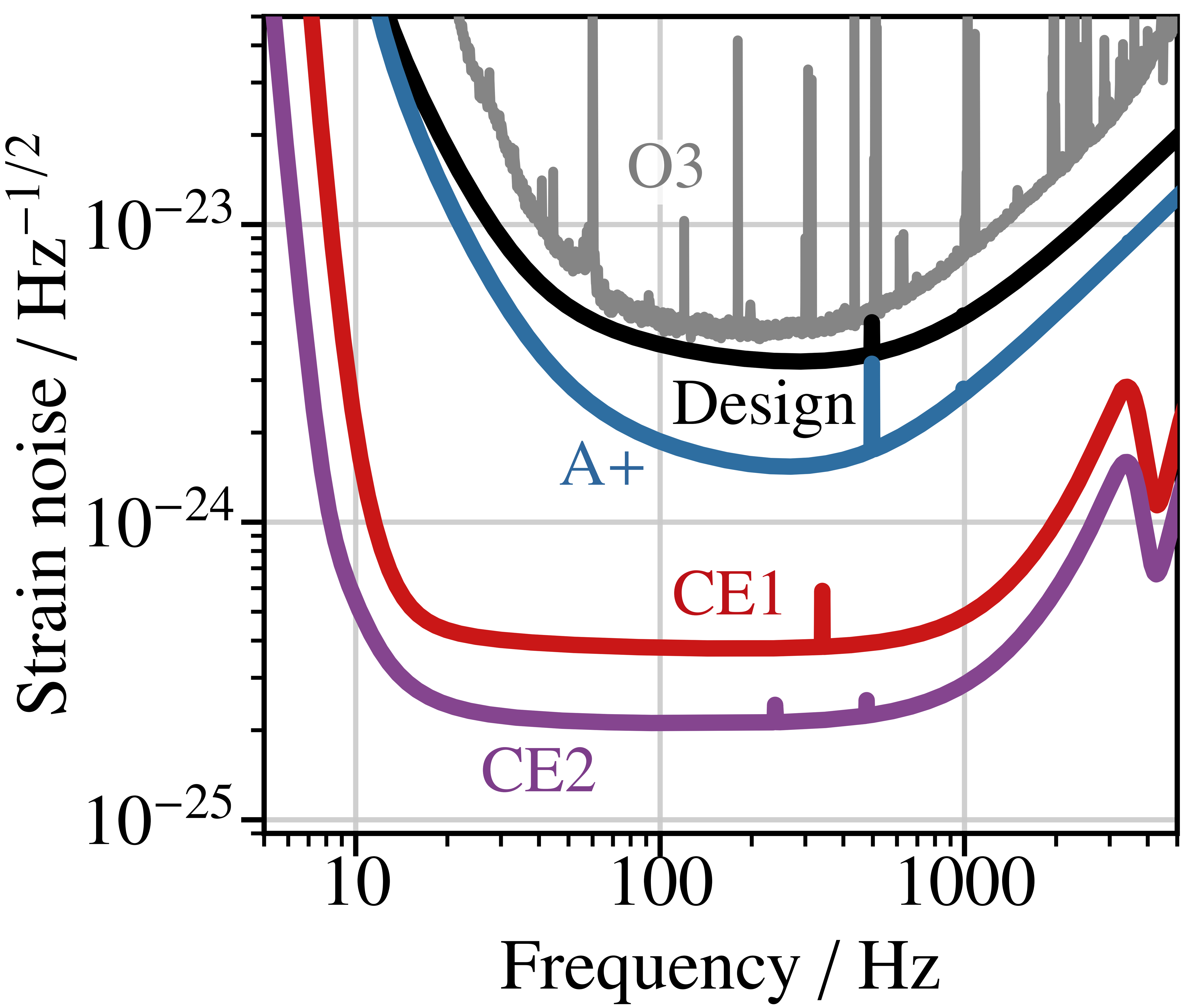

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$$S_h \sim \frac{4\hbar\omega_e}{P(k_e L B)^2} \quad \textbf{?}$$

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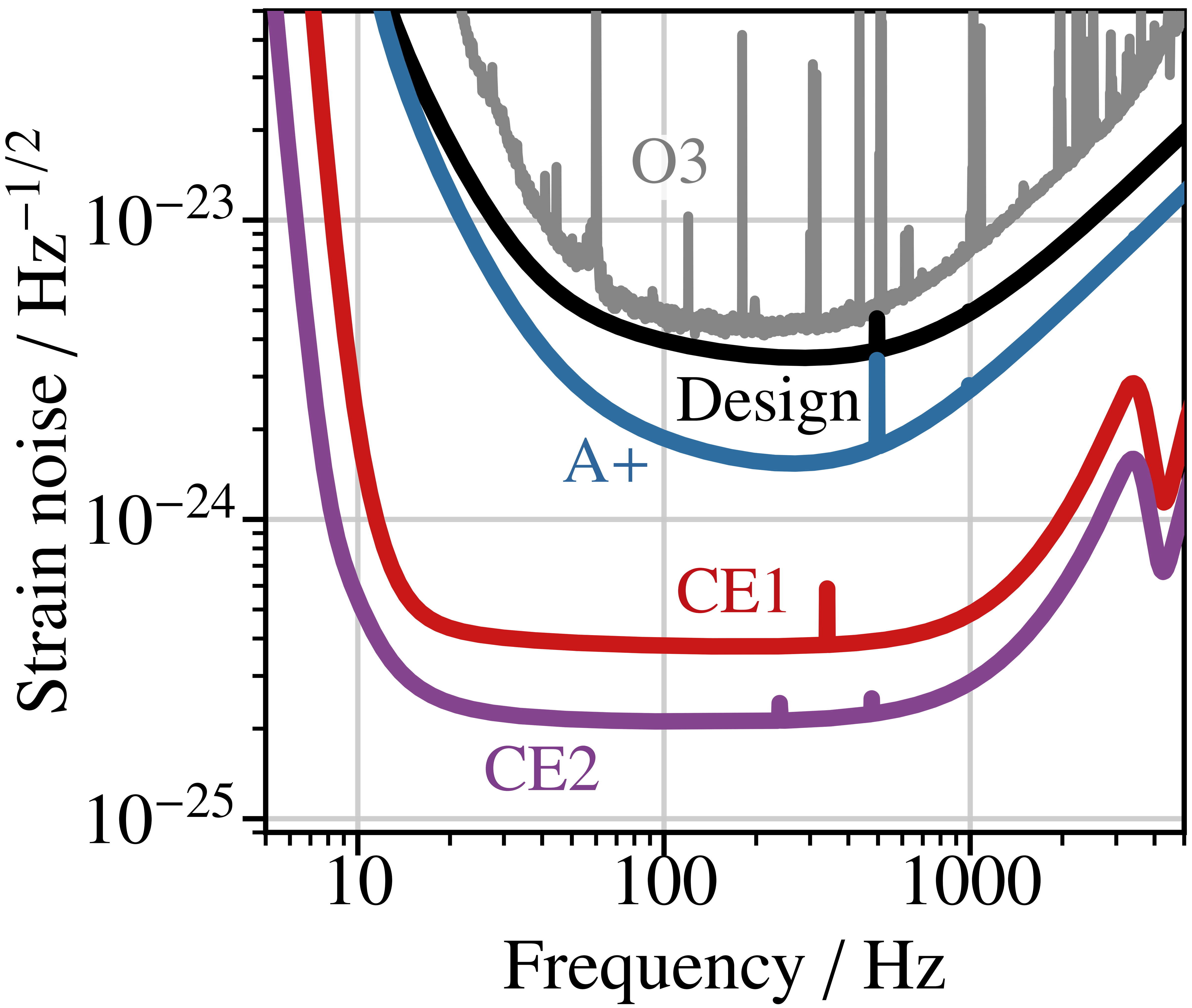

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$$h_{\rm rms}(\Delta f) = \sqrt{S_h \Delta f}$$

Expression we derived for the power requirement of the detector can be turned into an estimate of h_rms for a given power:

$$h_{\rm rms} \sim \sqrt{\frac{4 f \hbar \omega_e}{P(k_e L B)^2}}$$

$$S_h \sim \frac{4\hbar\omega_e}{P(k_e L B)^2} \;\; ?$$

Actually the correct answer except for a constant

To finish: Noise spectral density

Consider a function y(t) truncated to an interval of time T and its Fourier transform,

$$\tilde{y}(f) = \int_{-T/2}^{T/2} y(t)e^{i2\pi f t}dt,\ y_T(t) = \int_{-\infty}^{\infty} \tilde{y}(f)e^{-i2\pi f t}df$$

The integral over the square of a real function y(t) can be expressed as the following with the help of Parseval's theorem

$$\frac{1}{T}\int_{-T/2}^{T/2}[y(t)]^2 dt = \frac{2}{T}\int_0^{\infty} |\tilde{y}(f)|^2 df$$

Given this, if one defines the spectral density of y(t) as

$$S_y \equiv \lim_{T\to\infty}\frac{2}{T}\left|\int_{-T/2}^{T/2}(y(t)-\bar{y})e^{i2\pi f t}dt\right|^2$$

then the variance of y can be computed as:

$$\sigma_y = \int_0^{\infty} S_y(f)df$$

Want to know more?

https://astro-gr.org/online-course-gravitational-waves/

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Pygmalion OpenBSD WP Life Comments et al

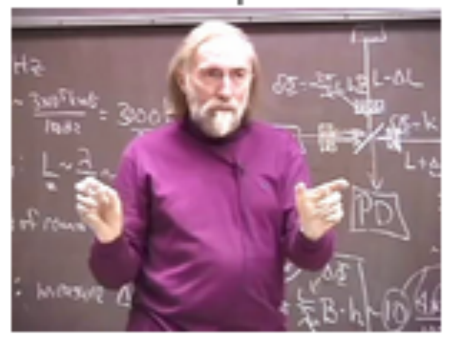
PART B: GRAVITATIONAL-WAVE DETECTORS

1- The Physics Underlying Earth-Based GW Interferometers – assignments and solutions

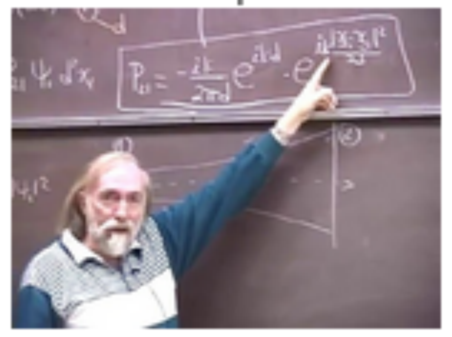
1. Idealized Interferometer: Conceptual design and crude analysis
2. General relativity: Proper reference frame of an accelerated observer
3. Optics
4. Statistical Physics: The theory of random processes

Lecturer Kip Thorne: **"The Physics Underlying Earth-Based GW Interferometers (1/4)"**

Lecturer Kip Thorne: **"The Physics Underlying Earth-Based GW Interferometers (2/4)"**



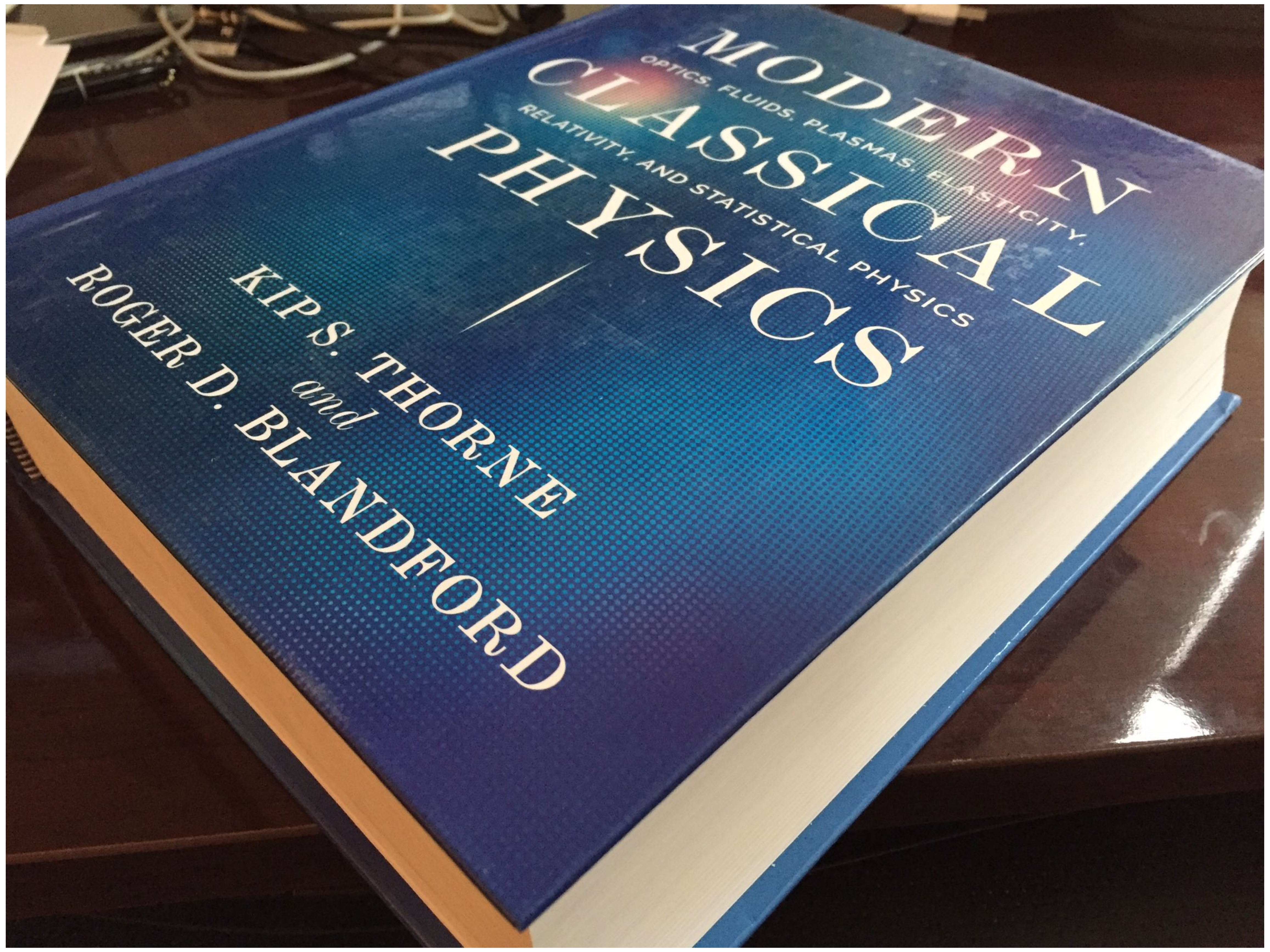
Lecturer Kip Thorne: **"The Physics Underlying Earth-Based GW Interferometers (3/4)"**



Lecturer Kip Thorne: **"The Physics Underlying Earth-Based GW Interferometers (4/4)"**

First four lectures on part B of this online course contain similar content to this lecture (plus information on physics of the lasers themselves). Following lectures in this online course go into a lot of additional detail.

Want to know more?



Specific chapters:

- 6.4.2 & 6.4.3: Spectral densities
- 6.74: Shot noise
- 9.4 & 9.5: Fabry-Perot interferometers
- 27.6: Detection of GWs

Free chapters are available online, from the notes of the course from which this book is based:

http://www.cns.gatech.edu/PHYS-4421/caltech136/index.html

The corresponding chapters in these notes are:

- 5.3 & 5.4: Spectral densities
- 5.5: Shot noise
- 8.4 & 8.5: Fabry-Perot interferometers
- 26.5: Detection of GWs, however it just says "Sorry: I have not yet written this section"

Want to know more?

Living Rev Relativ (2016) 19:3
DOI 10.1007/s41114-016-0002-8

CrossMark

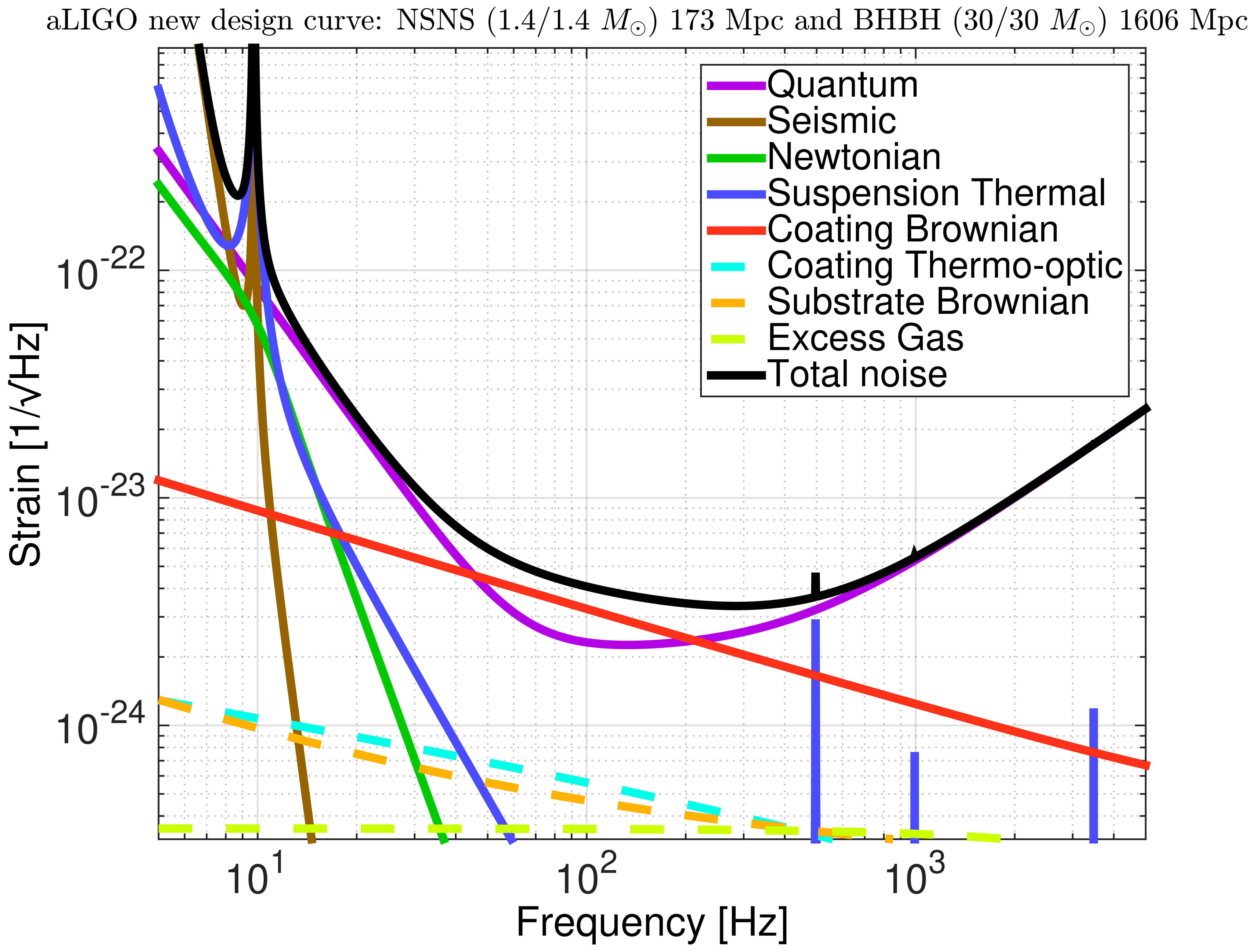
REVIEW ARTICLE

Interferometer techniques for gravitational-wave detection

Charlotte Bond[1] · Daniel Brown[1] · Andreas Freise[1] · Kenneth A. Strain[2]

217 page review paper with, as you might imagine, tons of detail. Discussion on the beam splitter was derived from section 2.4 here.

Challenge



We have derived a form for quantum shot noise that is independent of frequency:

$$S_h \sim \frac{\hbar \omega_e}{P(k_e L B)^2}$$

However quantum noise is dependent on frequency with

$$S_h \propto \begin{cases} f & \text{high } f \\ f^{-2} & \text{low } f \end{cases}$$

- Qualitatively describe what leads to this frequency dependency. Tip: low frequency behavior arises from radiation pressure noise modifying the interferometer arm lengths.
- Derive the scaling with frequency given above.