Image credit: LIGO/T. Pyle

Gravitational Wave Astrophysics

Pablo Marchant

Part 3: GWs from binaries

Outline

Today

- History of the field
- Types of detectors
- Types of sources
- Current state of the field
- Future advancements

12/5

- Ground based interferometers
- Production of GWs from compact object binaries

19/5

- Parameter estimation from observed compact object coalescences
- Astrophysics of observed GW sources

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Radiation from orbiting point masses

For an eccentric orbit, the time to merger can be computed from an integral expression. For a circular orbit the result is analytical:

$$t_d = \frac{a^4}{4\beta},\ \beta \equiv \frac{64}{5}\frac{G^3 m_1 m_2 (m_1+m_2)}{c^5}$$

Using Kepler's third law, this can be expressed in terms of the orbital period and a combination of the masses called the chirp mass

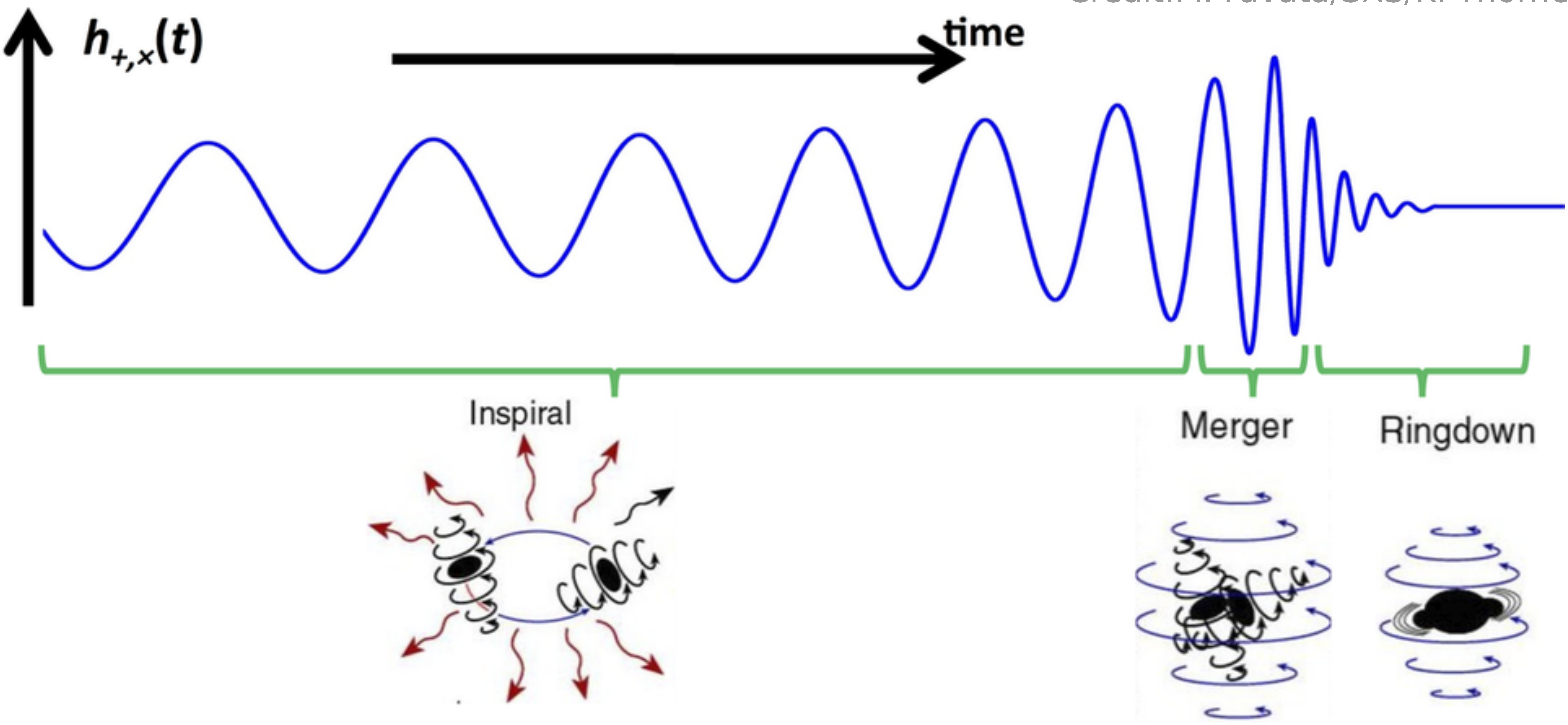
$$t_d = 7.4\ [\mathrm{Gyr}] \left(\frac{P}{12\ [\mathrm{h}]}\right)^{8/3} \left(\frac{\mathcal{M}}{M_\odot}\right)^{-5/3},\ \mathcal{M} \equiv \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

$$\text{for } m_1 = m_2,\ \mathcal{M} \simeq 0.87 m_1$$

Does nature provide such massive and compact binaries?

Producing a waveform

Credit:M. Favata/SXS/K. Thorne



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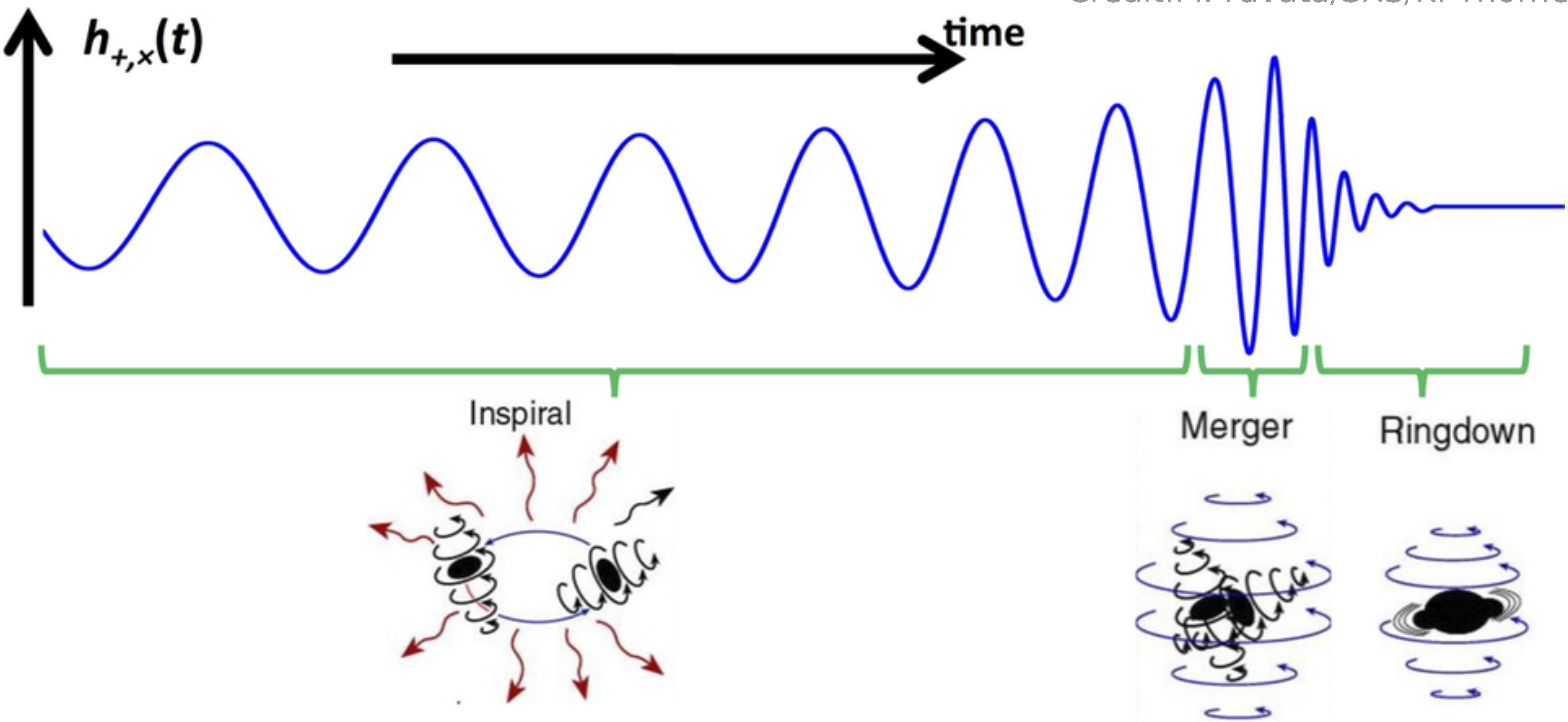

Post-newtonian theory

Numerical relativity

Perturbation theory

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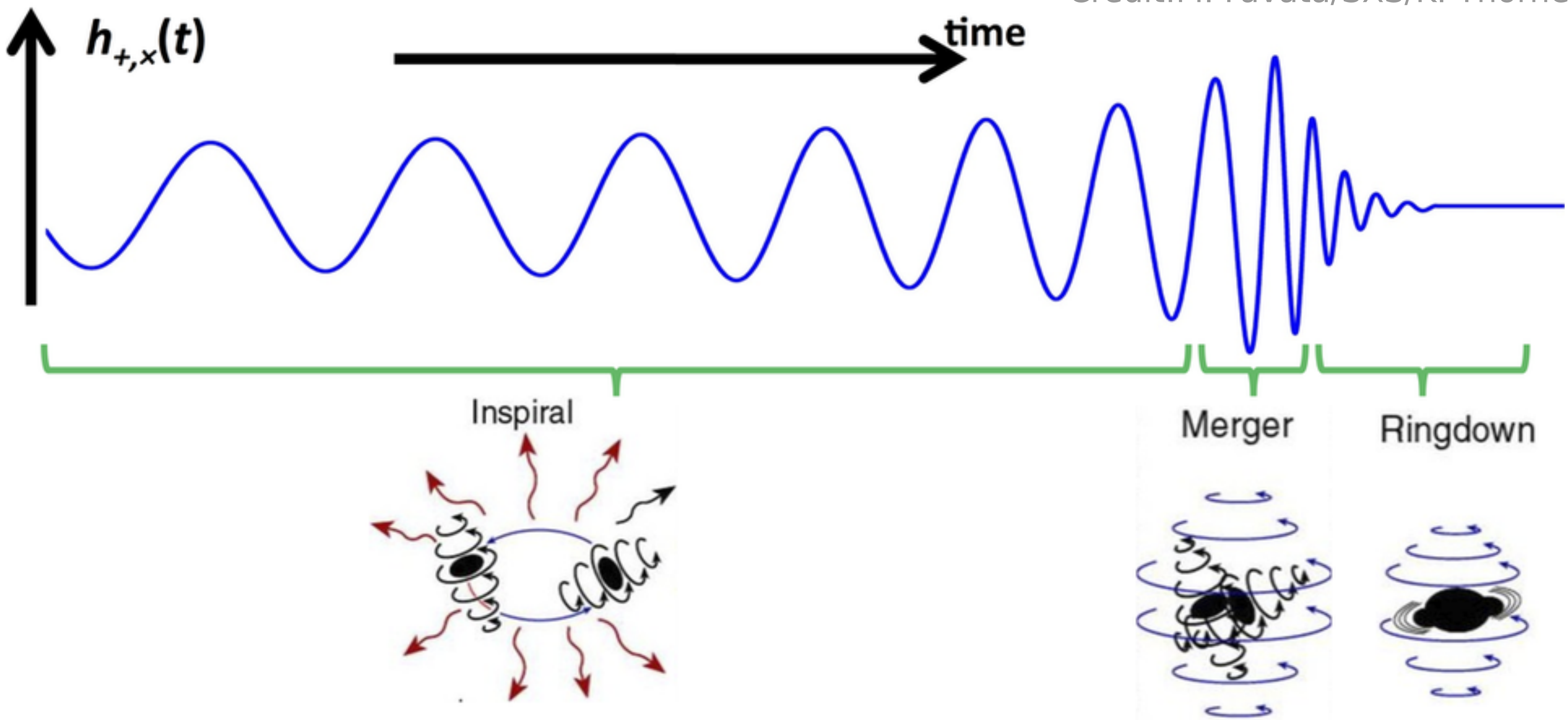

Post-newtonian theory

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https://www.youtube.com/watch?v=p647WrQd684

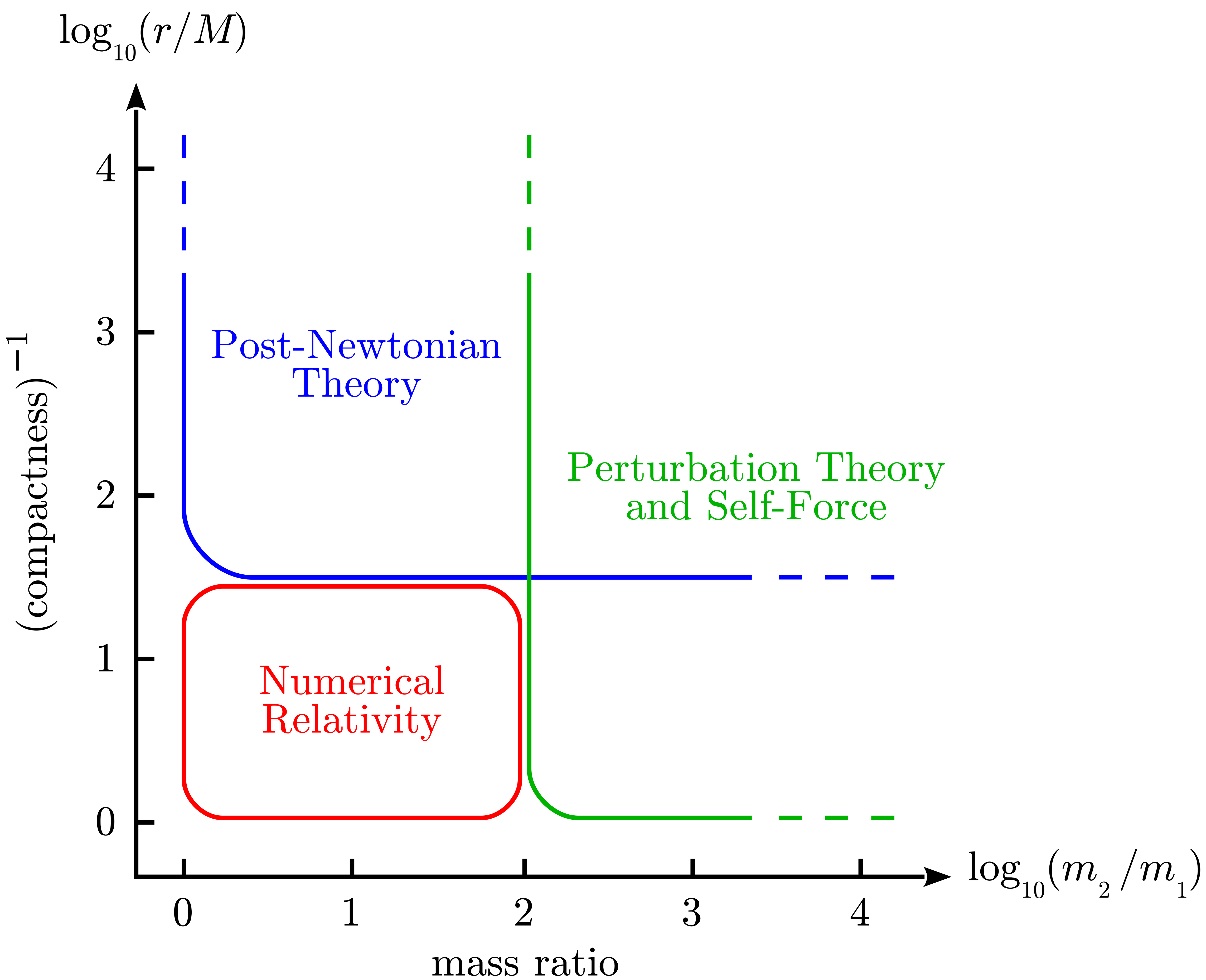
Producing a waveform

- Different parts of the parameter space require different techniques. (although NR in principle can cover all)

- NS succeded only recently in doing a complete merger simulation. (Pretorius 2005).

- PN theory, expansion in terms of (v/c)^2 factors, valid for v<<c.

- Lowest order of PN theory with gravitational radiation is the so-called 2.5 PN order (quadrupole radiation).



Le Tiec (2014), astro-ph: 1408.5505

Example of the complexity of an extreme mass ratio inspiral https://www.youtube.com/watch?v=bqFHe7CM99g

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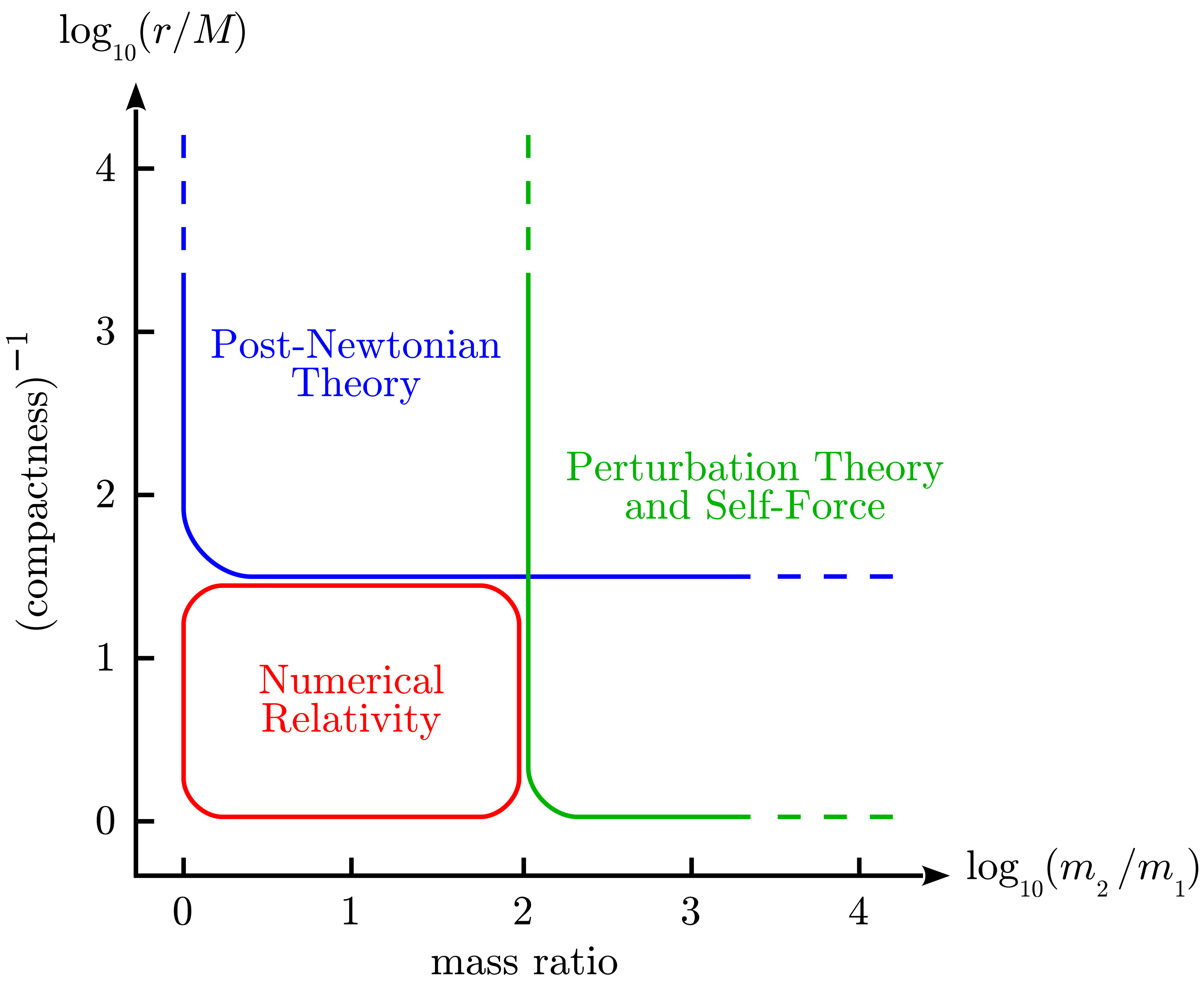

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I will only discuss the impact of quadrupole radiation here!

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Quadrupole moment

The Newtonian potential from a mass distribution can be computed as:

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$$\frac{1}{|\mathbf{x}-\mathbf{x}'|} = \frac{1}{r} + \frac{x^j x^{j'}}{r^3} + \frac{x^j x^k (3x^{j'} x^{k'} - r'^2 \delta_{jk})}{2r^5} + \ldots,\ r = |\mathbf{x}|$$

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From which the potential can be expressed as a multipolar expansion:

$$\Phi(\mathbf{x}) = \frac{M}{r} + \frac{\mathbf{x}\cdot\mathbf{d}}{r^3} + \frac{3x^j x^k \mathcal{I}_{jk}}{2r^5} + \ldots$$

$$M = \int \rho dV_x, \; \mathbf{d} = \int \rho \mathbf{x} dV_x, \; \mathcal{I}_{jk} = \int \rho(x^j x^k - r^2 \delta_{jk}/3)$$

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Radiation can also be produced by moments of the mass "current" (ie. momentum). Current quadrupole radiation is the first term that contributes, but is a factor v/c smaller than the mass quadrupole.

Quadrupole radiation

Exercise 1

- Demonstrate that the second time derivative of the dipole moment is zero for a closed system.
- Demonstrate also that the second time derivative of the current dipole,

$$\int \rho(\mathbf{x})\mathbf{x} \times \mathbf{v} dV$$

is zero in a closed system.
- Show that a spherical mass distribution has a zero quadrupole moment. What about higher moments?

Grav ... ould
exhi ... ves.

M ... $/3)$

Ma

o!

Radi ... e.
mom ... ibutes,
but is

Quadrupole radiation

Without going into too much detail, what we are interested in is the gravitational wave signal induced. In an informal notation, one has that the strain produced by the variation of the mass quadrupole is:

$$h_+ \sim h_- \sim \frac{G}{c^4}\frac{\ddot{\mathcal{I}}_{jk}(t-r/c)}{r}$$

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Exercise 2

Imagine we want (for some malicious reason) to produce a fake GW signal in the Virgo detector in Italy. Rather than going there and stomping on the ground we decide to tie to masses with a 1 meter cord and make them rotate at 100 Hz. What masses would we require?

Last … GW
radia…

$$M \sim \quad \ldots 0^{20}\ [\mathrm{m}]$$

$$h \sim 10^{-25}$$

Binary systems

$$h_+ \sim h_- \sim \frac{G}{c^4}\frac{\ddot{\mathcal{I}}_{jk}(t-r/c)}{r}$$

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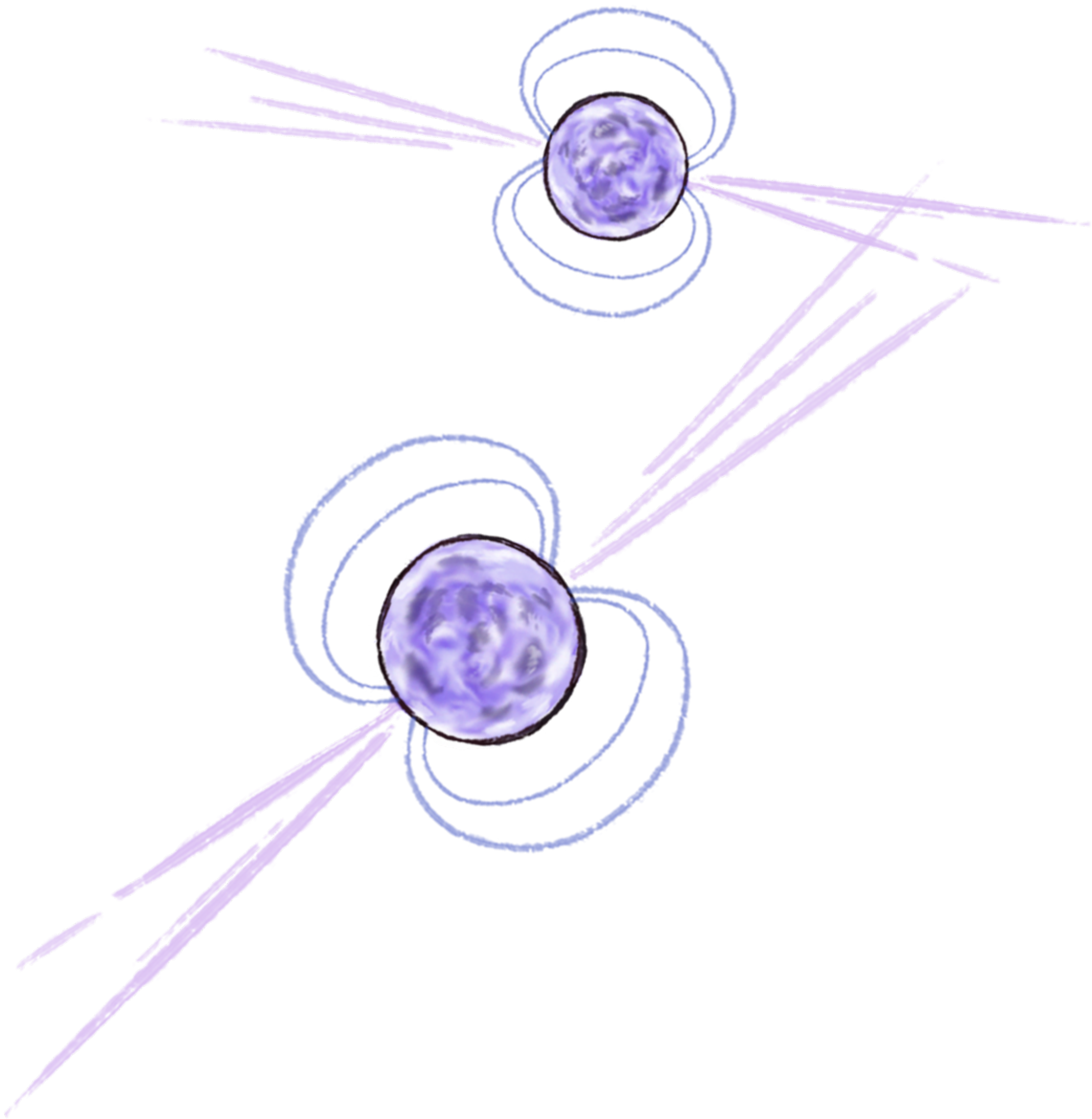
In a binary system T and L are not arbitrary, but connected through Kepler's third law,

$$\Omega = \frac{2\pi}{P} = \sqrt{G(M_1+M_2)/a^3}$$

Credit: Sara Pinilla



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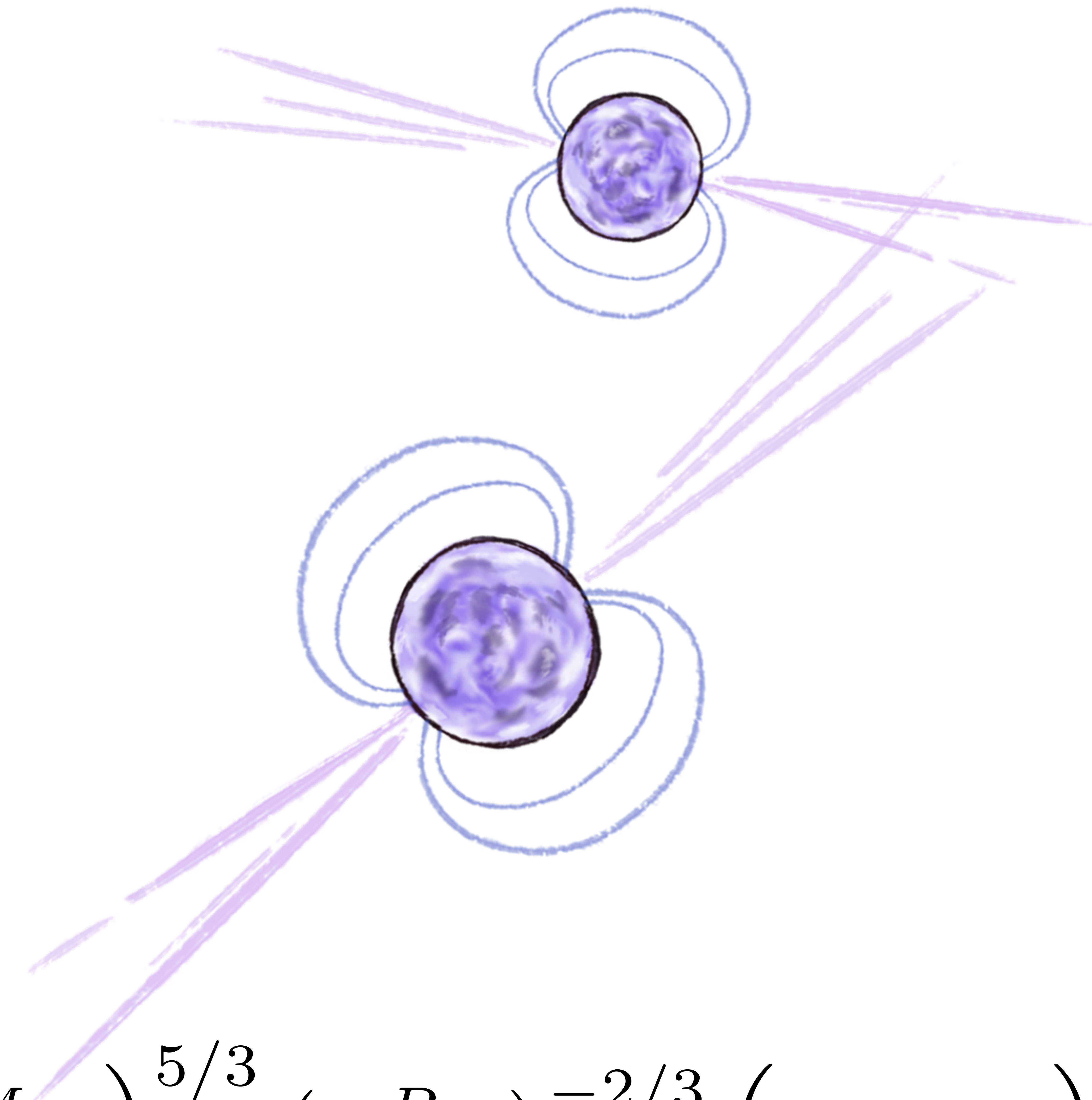

$$h \sim \frac{G^{5/3}}{c^4}\frac{M^{5/3}}{rP^{2/3}} \simeq 3\times 10^{-21}\left(\frac{M}{60\ M_\odot}\right)^{5/3}\left(\frac{P}{0.01\ \mathrm{s}}\right)^{-2/3}\left(\frac{r}{100\ \mathrm{Mpc}}\right)^{-1}$$

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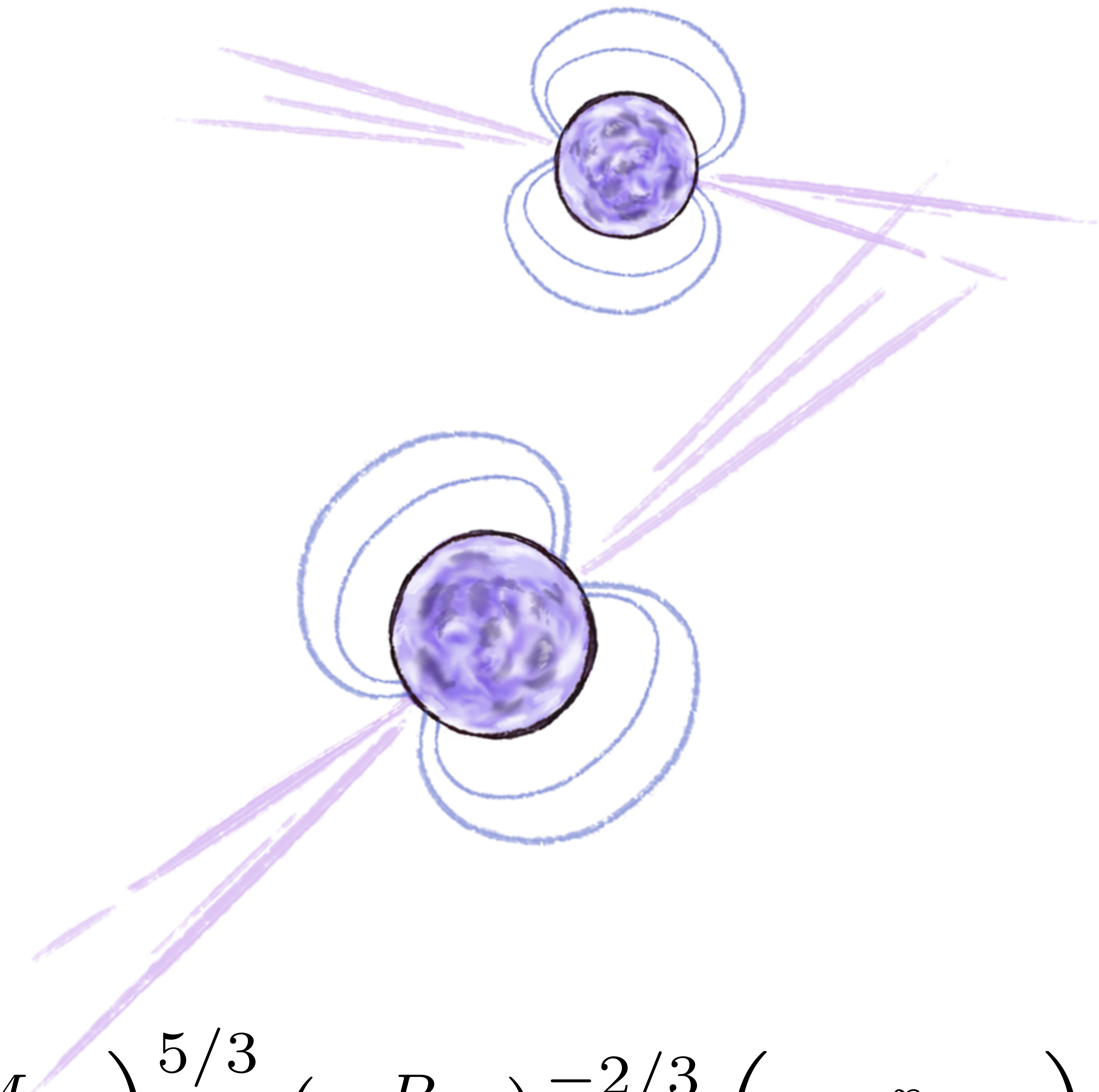

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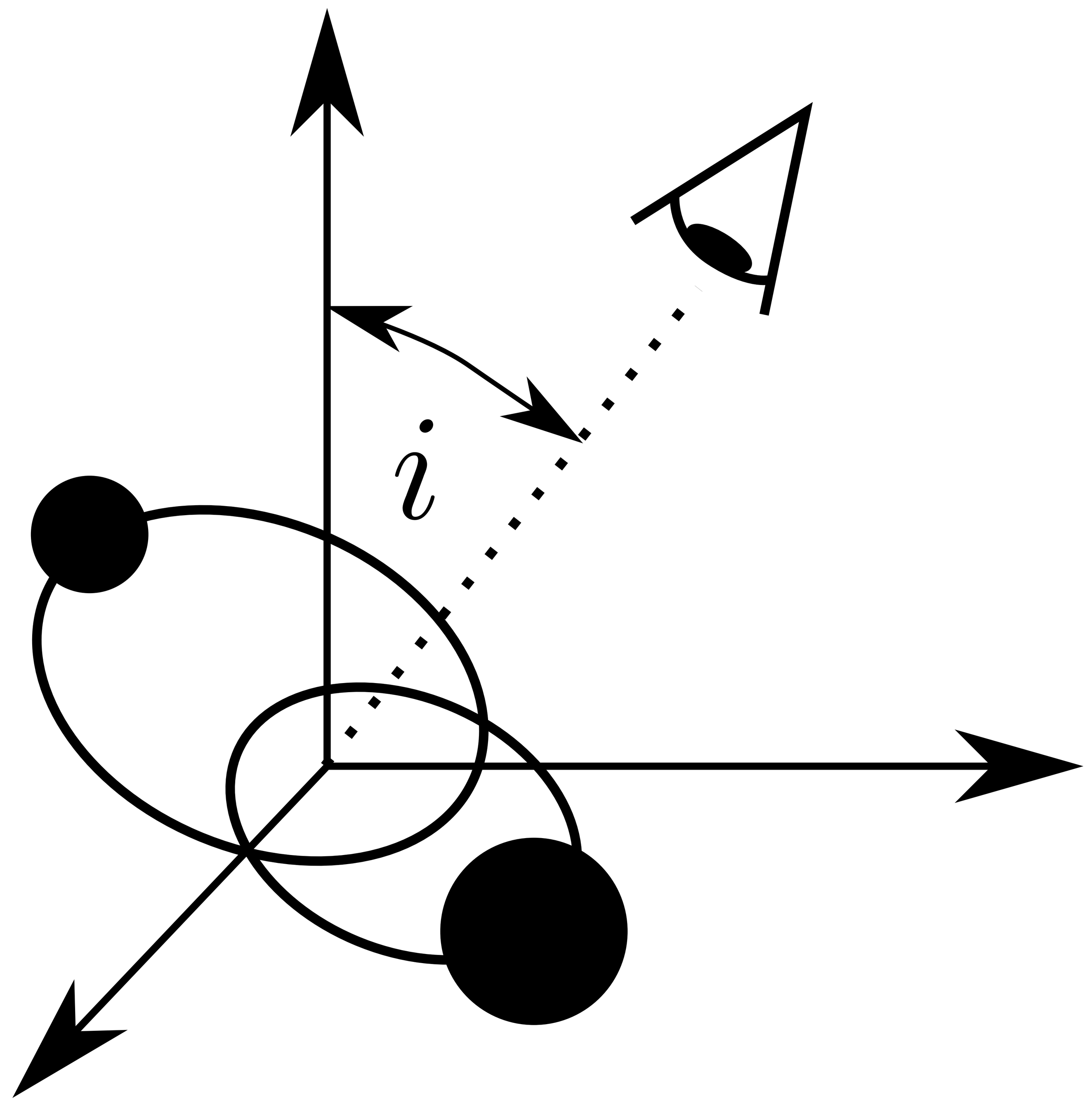
Of course, not really Keplerian at 100 Hz

Circular binary system



Let's write down the exact solution for h for a circular binary. Consider the system is observed at an inclination i.

$$M = m_1 + m_2,\ \mu = \frac{m_1 m_2}{m_1 + m_2}$$

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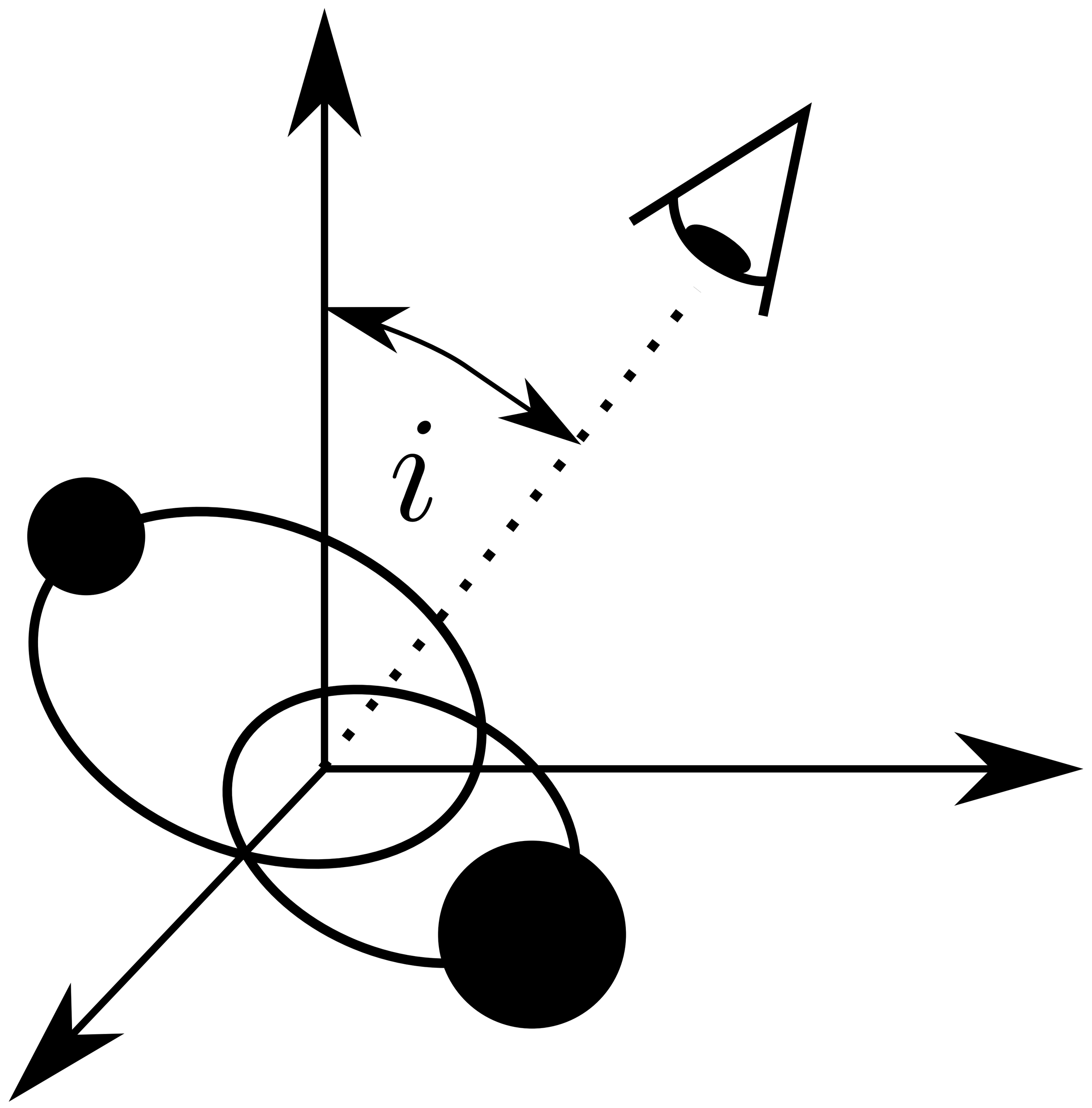

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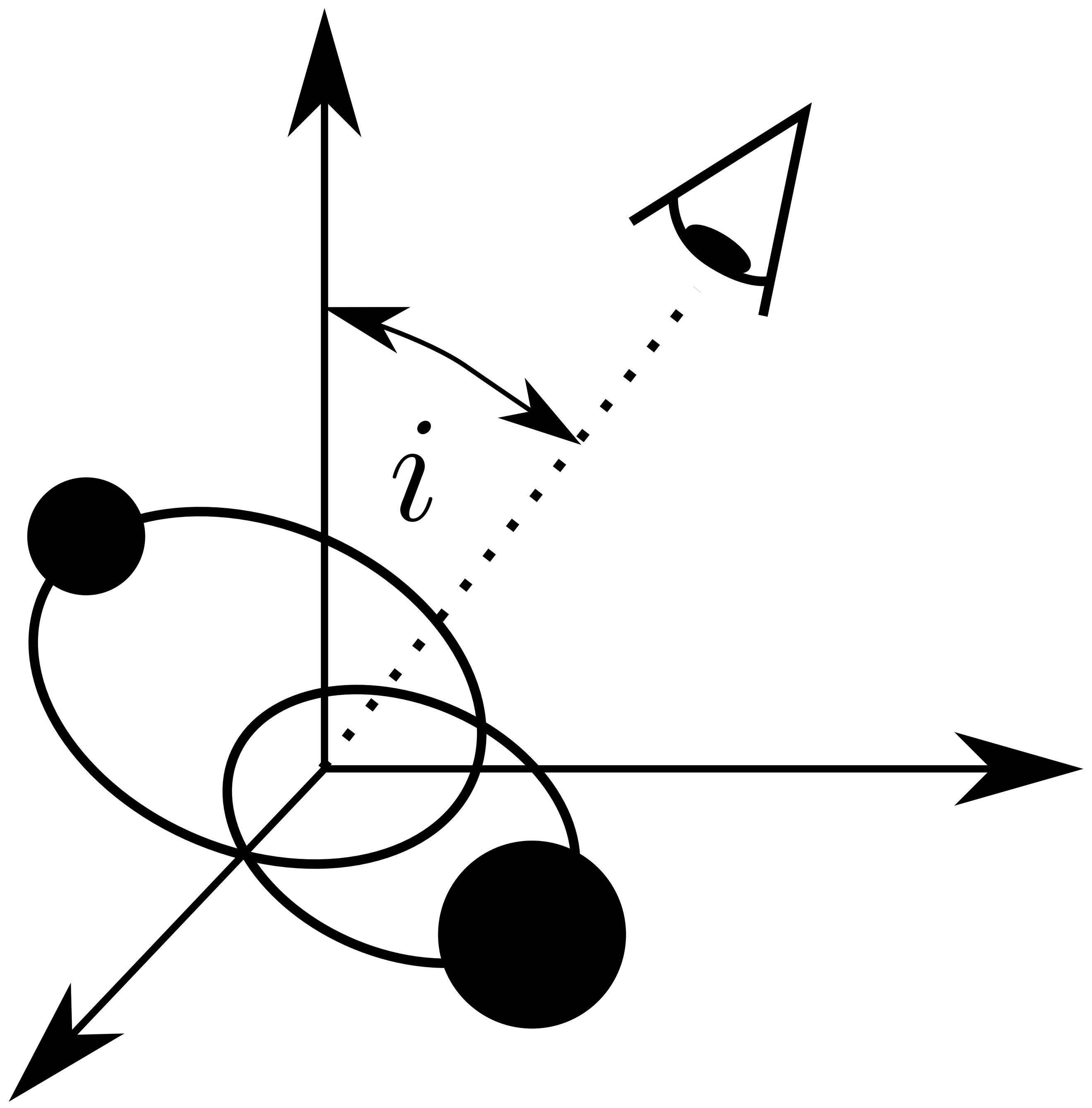

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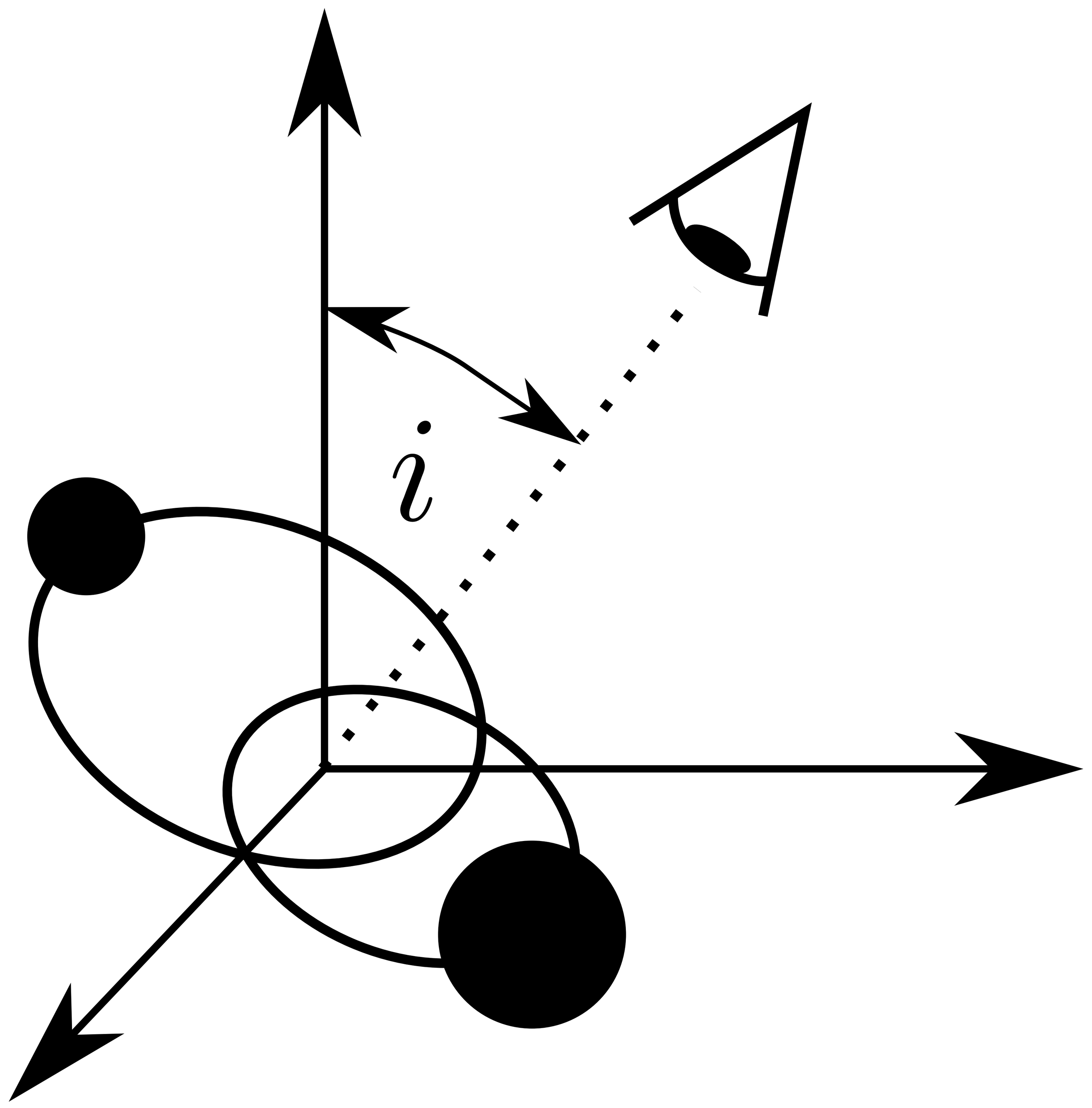

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polarizations are shifted by a quarter phase, circular polarization for i=0!

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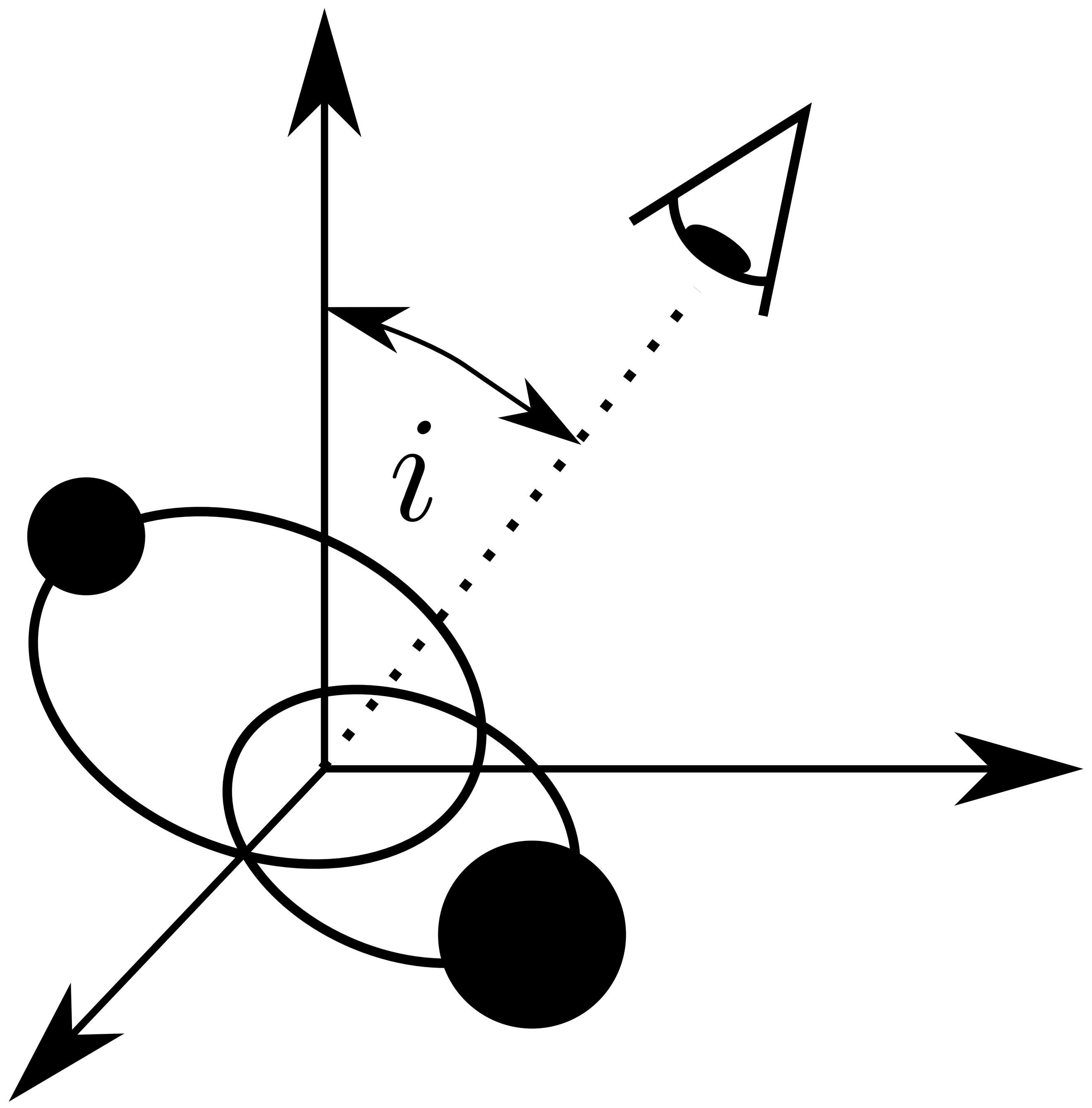

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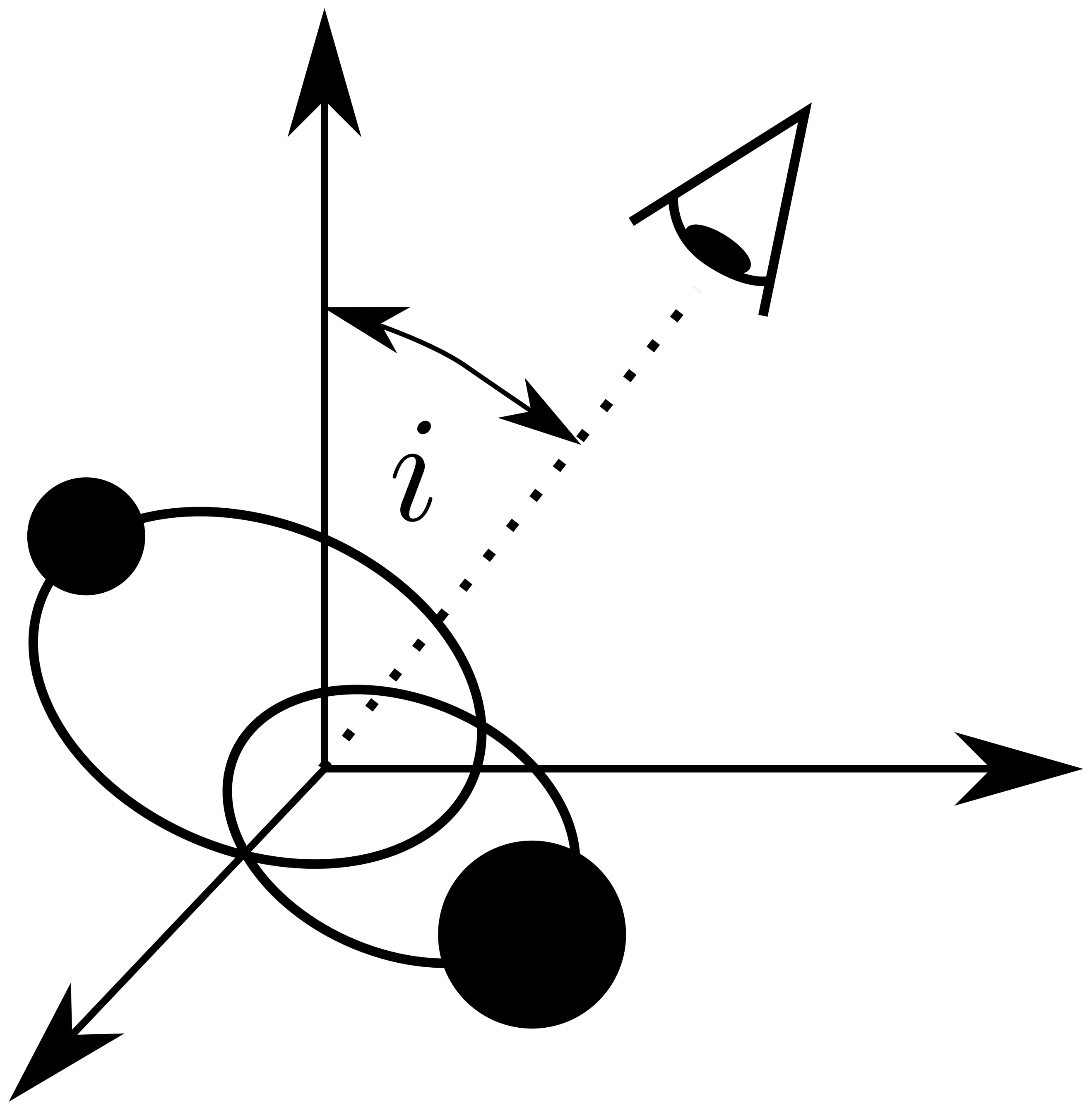

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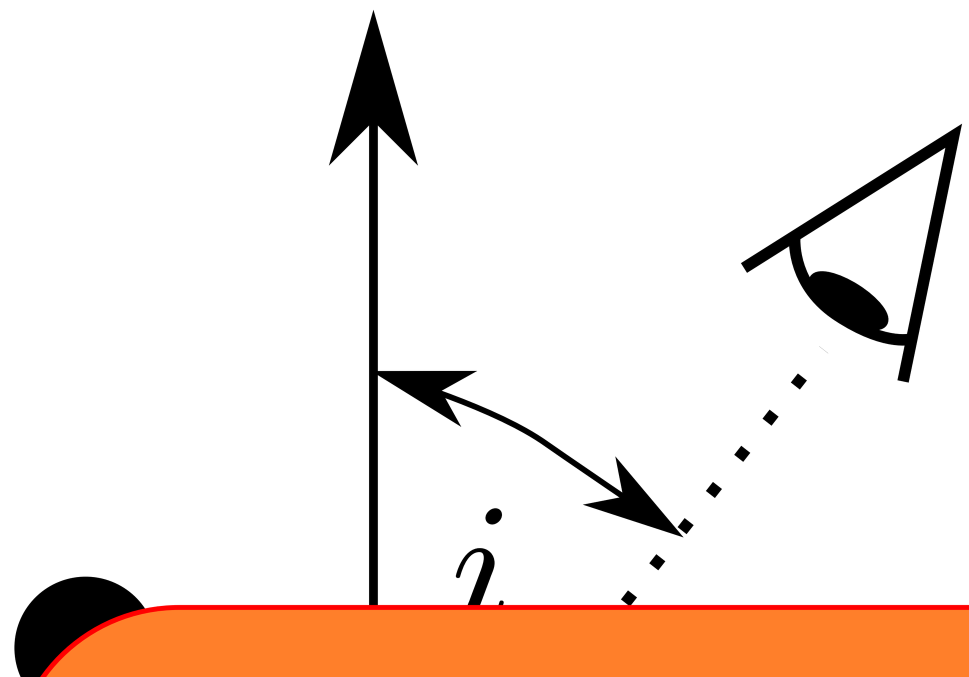
https://www.youtube.com/watch?v=Y6tSFk5ESAo



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m_2 / $-m_2$

Mea… con… divi… …orbital!

h_+ … $- \phi)]$

Exercise 3

From the mass quadrupole moment determine that the orbital period of a GW would be half that of the binary system. To do this consider a coordinate frame on the center of mass and see how the mass quadrupole changes after half an orbit.

$$h_- = -4\cos\theta \frac{G^{5/3}}{c^4}\frac{\mu(M\Omega)^{2/3}}{r}\sin[2(\Omega t - \Omega r/c - \phi)]$$

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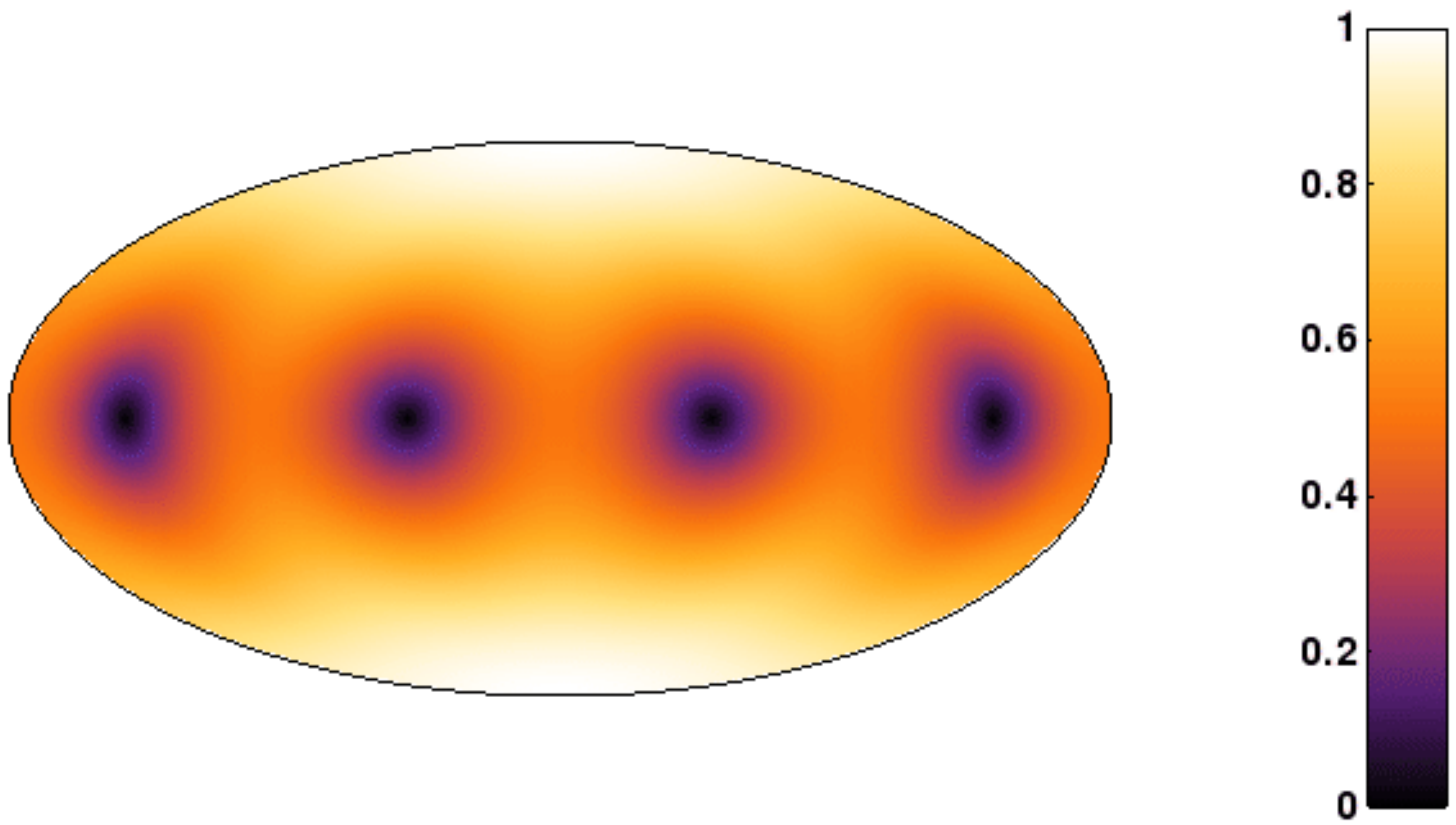
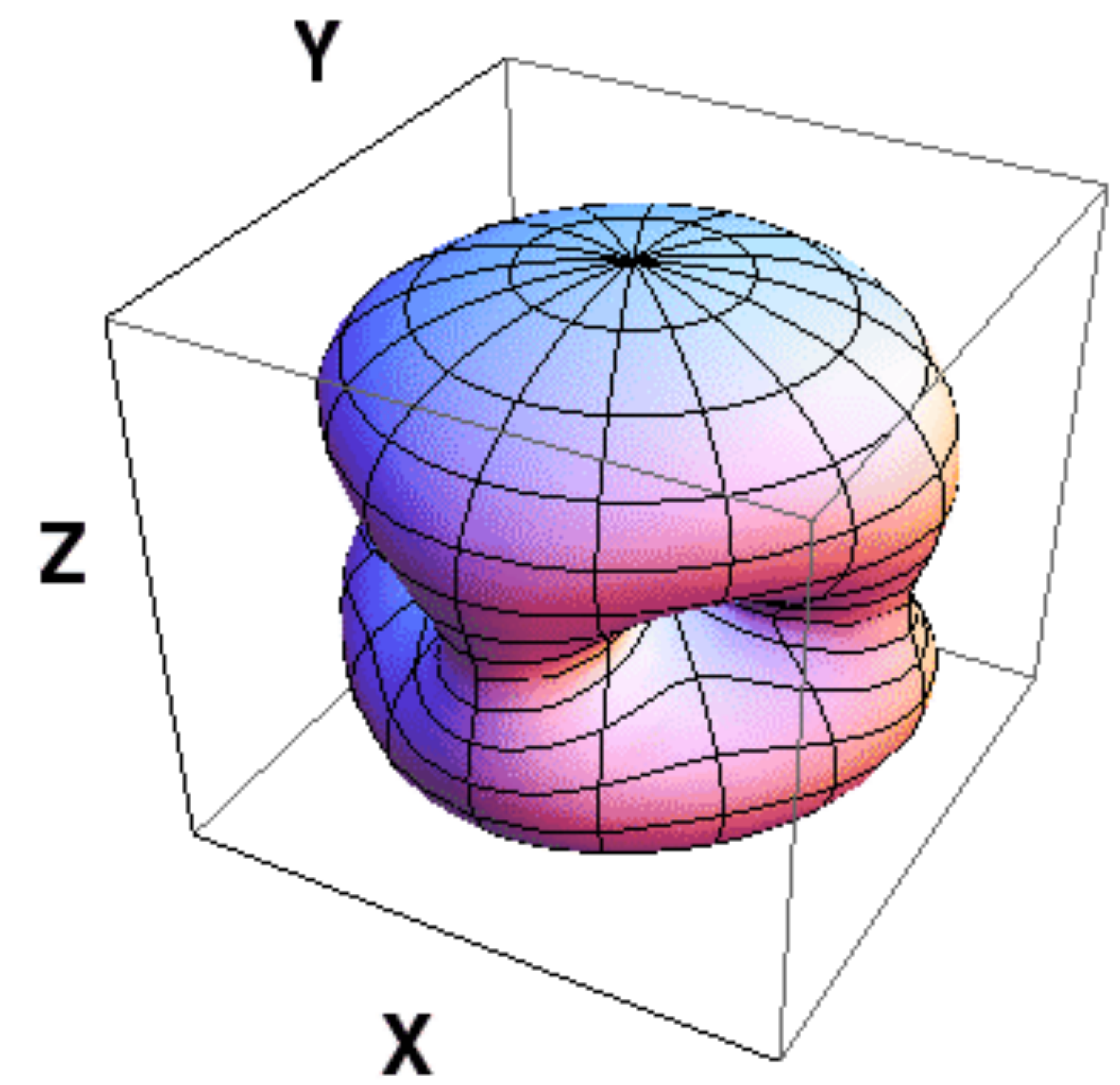
Brief note on interferometers

Although interferometers are not pointed at a specific location, the have different sensitivities to waves coming from different sky locations,





Moore, Cole & Berry (2014)

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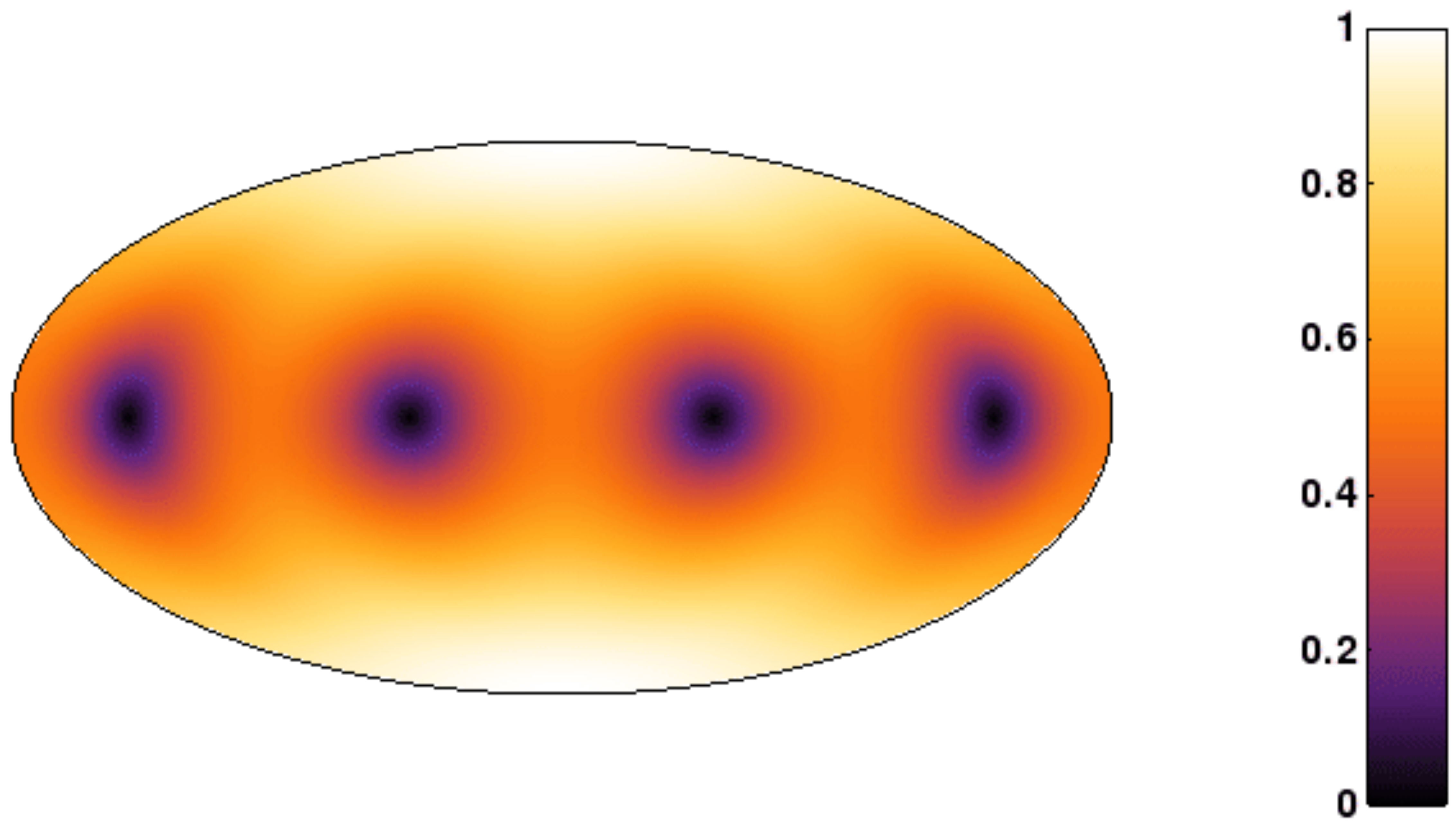
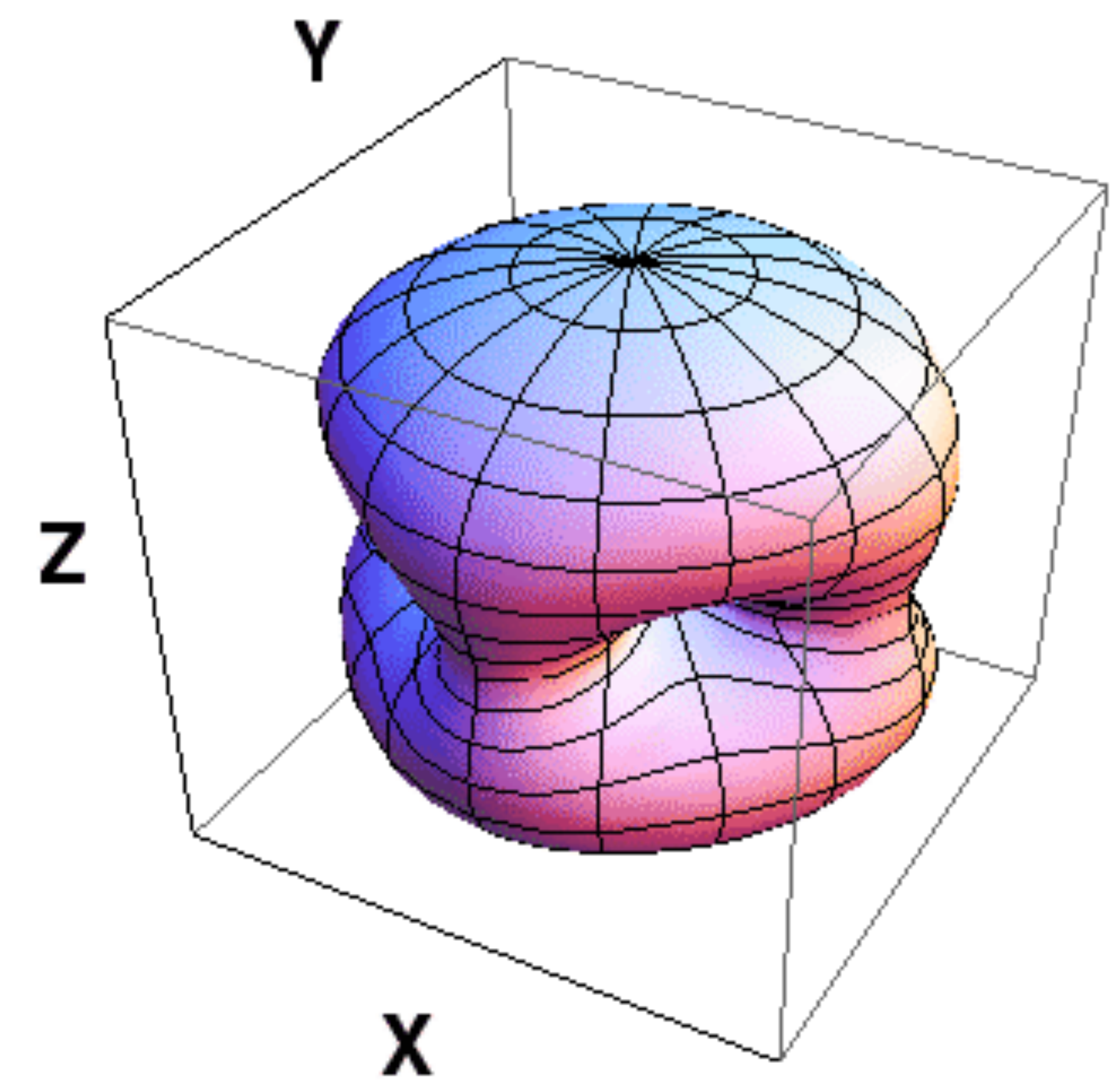


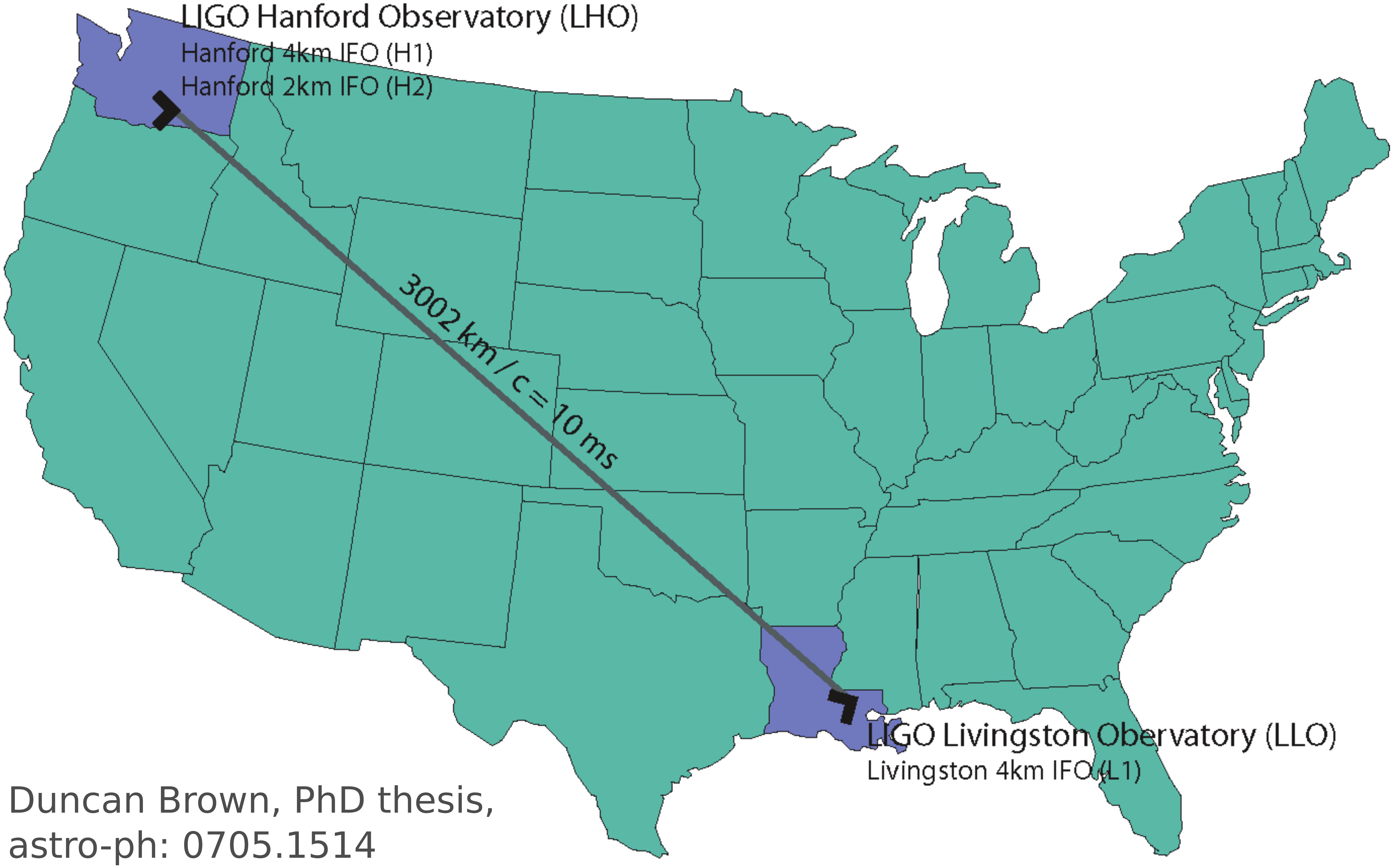


Moore, Cole & Berry (2014)

Additional, the orientation of a binary affects sensitivity. From the equations on the previous slide, one can see that the ideal alignment is the observation of a system with i=0, seen directly above or below the detector.

Brief note on interferometers

Since an interferometer is sensitive to only one polarization, LIGO detectors are placed at a slight angle mismatch.



Duncan Brown, PhD thesis,
astro-ph: 0705.1514

Chirp mass

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But one does not need to wait for a merger to happen to measure the chirp mass. It can be computed from the frequency and its time derivative:

$$\mathcal{M} = \frac{c^3}{G} \left(\frac{5}{96}\pi^{-8/3} f^{-11/3} \dot{f}\right)^{3/5}$$

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As we discussed last week, the time to merger for a compact object binary is a function of period and the chirp mass

$$t_d = 7.4\ [\mathrm{Gyr}] \left(\frac{P}{12\ [\mathrm{h}]}\right)^{8/3} \left(\frac{\mathcal{M}}{M_\odot}\right)^{-5/3},\ \mathcal{M} \equiv \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

But one does not need to wait for a merger to happen to measure the chirp mass. It can be computed from the frequency and its time derivative:

$$\mathcal{M} = \frac{c^3}{G} \left(\frac{5}{96}\pi^{-8/3} f^{-11/3} \dot{f}\right)^{3/5}$$

You might have not noticed, but we saw the chirp mass a few slides ago. In the strain amplitude the mass comes in the form:

$$\mu M^{2/3} = \frac{m_1 m_2}{(m_1+M_2)^{1/3}} = \mathcal{M}^{3/5}$$

Chirp mass

As we discussed last week, the time to merger for a compact object binary is a function of period and the chirp mass

Take home messages

- Measuring the period and period derivative of a slow motion binary gives its chirp mass.
- Measuring the degree of circular polarization gives a measure of its inclination.
- Measuring the intensity of the wave, coupled with the chirp mass, gives the distance to the source.

$$\mu M^{2/3} = \frac{m_1 m_2}{(m_1+M_2)^{1/3}} = \mathcal{M}^{3/5}$$

Want to know more?

https://astro-gr.org/online-course-gravitational-waves/

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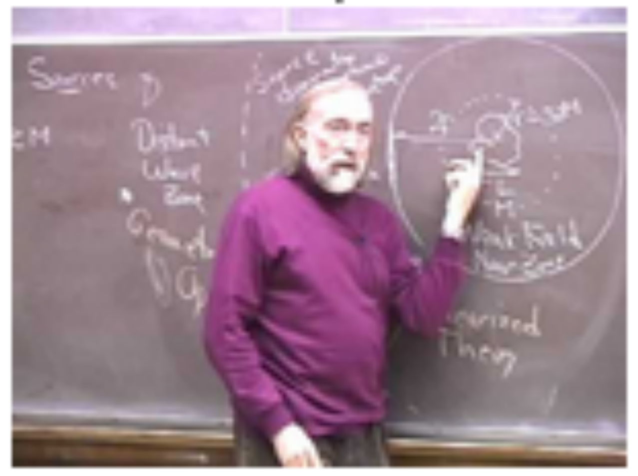
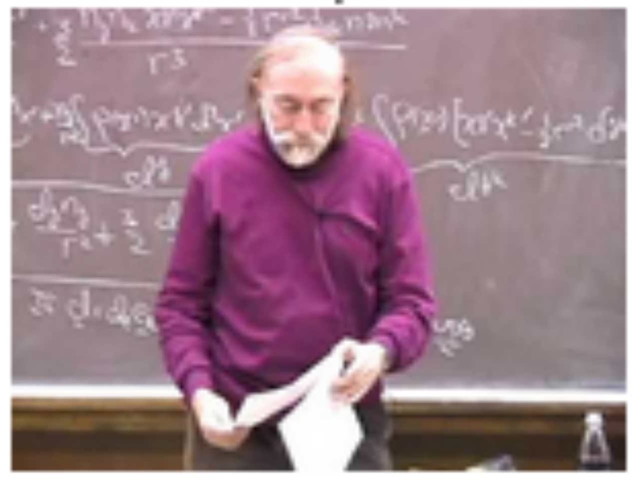
5- Generation of GWs by Slow-Motion Sources in Curved Spacetime

1. Strong-field region, weak-field near zone, local wave zone, distant wave zone
2. Multipolar expansions of metric perturbation in weak-field near zone and local wave zone
3. Application to a binary star system with circular orbit

Lecturer Kip Thorne: **"Generation of GWs by Slow-Motion Sources in Curved Spacetime (1/2)"**

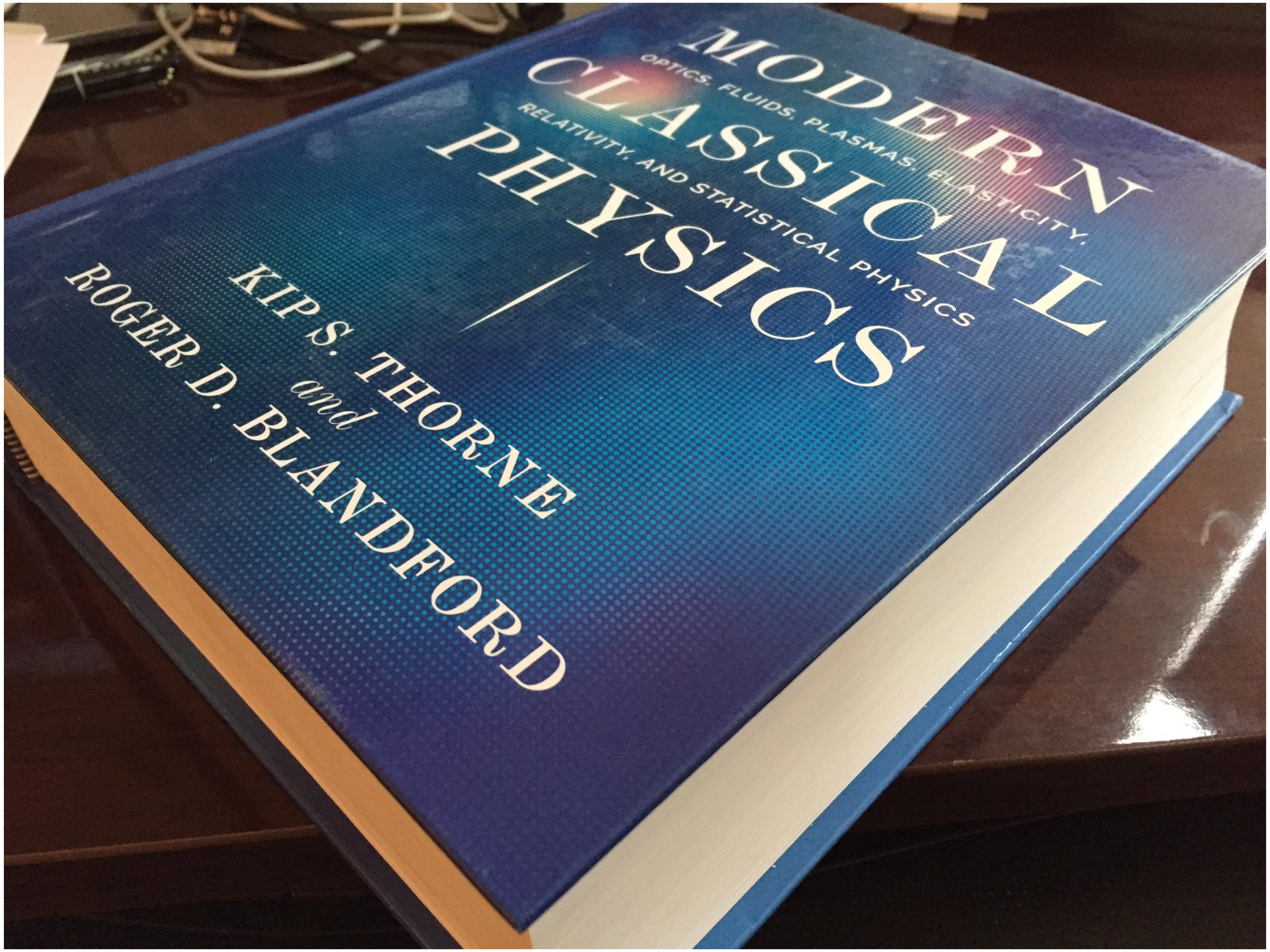


Lecturer Kip Thorne: **"Generation of GWs by Slow-Motion Sources in Curved Spacetime (2/2)"**



Includes GR derivations, various lectures on things such as post-newtonian approximations and numerical relativity.

Want to know more?



Chapter 27.5 on generation of gravitational waves